

Afterword: After How Comes What

In a conversation about social science, Marvin Minsky - who was famous for his caustically insightful witticisms - once said: "Anything that calls itself science isn't." Socratically stung by this remark, over the years I have built up a collection of features a would-be new science might borrow from disciplines that do not proclaim their science-ness in their names. Keith Sawyer positions this volume as number two after *How People Learn* in the evolution of a new science, the learning sciences (see Preface, this volume]. Perhaps a sample from my collection, presented as an afterword to number two, might prefigure a foreword to number three.

A Sense of the Fundamental

Consider the speeds, of light, of sound, and of the bullet train. Each is *important* in its context but physicists would agree that the first has the special status of being *fundamental*. This volume treats many *important* ideas about learning. But our still embryonic science has not yet developed a consensus about which ideas are *fundamental*.

At critical times the mature sciences have identified critical problems whose solutions would spawn theories with fundamental consequences: Is light a particle or a wave? Is there a universal decision procedure for mathematics? What is the genetic code? From time to time one or another theory of learning has enjoyed a heady period of seeming to have a similarly fundamental status - examples include behaviorism, Piaget's theories, and information processing models. But so far all have failed to generate either lasting consensus or transformational advances in practice. The learning sciences will escape Minsky's sting when new contenders for fundamental status emerge.

This essay is guided by a perception that this emergence is inhibited by the narrowness of the boundaries that learning sciences has set for itself. My title expresses one facet of this perception: educational psychology has often focused on how people learn knowledge entities that are given independently of it, while the deepest source of new ideas may be the exercise of inventing new entities. As this possibility becomes apparent in our community, I anticipate a widening of focus from *how* people learn to include more study of *what* they learn. The importance of this issue to the learning sciences is reflected in the title of Part 3 of this handbook, "The Nature of Knowledge" (also see Sawyer's introduction). Could there be a scientific basis for deciding what children should know? What properties of knowledge make it more or less learnable or more or less able to facilitate other learning?

A Thought Experiment

Einstein's famous thought experiment of being in an elevator whose cable had snapped made its point as solidly as equations or experimental observation. I illustrate my belief that a more developed culture of *properly disciplined* thought experiments would enrich thinking about learning by using a thought experiment.

Imagine that learning scientists existed in the days when numbers were manipulated using Roman numerals. Imagine that because only a small number of people could do multiplication, economic progress was very slow, and that learning scientists were funded to mobilize all the great ideas in *How People Learn* to remedy the situation. Undoubtedly better teaching would increase the number of people capable of performing the complex art of multiplication. But something else did this far more effectively: the invention of Arabic arithmetic, which turned the formerly esoteric skill of multiplication into one of the basics.

The question to ponder is how the invention of Arabic arithmetic is related to learning sciences. A simplistic answer is: *not at all, it belongs to mathematics*. Indeed, historically, Arabic numerals were not invented with an educational intent. *But they could have been*. And would that bring it within the scope of our science? Or ponder this: Even if learning scientists considered that the invention of representations for numbers belonged to another discipline, was it not their duty -

before accepting funding to study how to teach Roman numerals better - to perform due diligence to determine whether another way existed? And how could they do this without making part of their science the study of alternative structures of disciplinary knowledge?

Liberating Mathematics from Math!

These questions are relevant to thinking about the boundaries of the learning sciences but they barely touch the surface. To go deeper, consider the opinion of Steve Pinker about why language is easier to learn than mathematics:

On evolutionary grounds it would be surprising if children were mentally equipped for school mathematics. These tools were invented recently in history and in only a few cultures, too late and too local to stamp the human genome.

Stated simplistically, Pinker's Chomskian position is that language learning has become innate because language is old enough to influence the emergence of genes that support it, whereas algebra has not been around long enough. I propose an alternative theory which allows a more constructive role for learning sciences: Language did not stamp the genome, the genome stamped language. Language molded itself, as it developed, to genetic tools already there. The reason algebra is less well aligned with genetic tools is that it was *not allowed* to align itself: it was *made* by mathematicians for their own purposes while language *developed* without the intervention of linguists.

This theory suggests how one might make an entity that would stand to algebra as Arabic arithmetic to Roman arithmetic: a different way (possibly only useful to learners) of achieving the same functional ends. *Creating such entities would be doing artificially for elementary mathematics what a natural process did for language.*²

This idea would not long ago have been of only very abstract interest. It is brought down to earth by the presence of digital technologies, combined with theoretical observations - such as noting that computer programming languages share functions and structures with both algebra and natural language. Their language-like side enters in two ways. The first is the now-well-established fact that programming languages such as Logo, Squeak, Boxer, and ToonTalk (Kafai, this volume; Noss & Hoyles, this volume) permit very young concrete-minded children to command computers to perform actions of personal interest. The second, which is more often missed, is seen in the design criterion for Logo to make its version of "variable" also be a version of "pronoun," thus reducing the cognitive distance between algebra and language and making it plausible that both could draw on the same genetic tools. Thus, in what might be a theoretically important sense, saying the word "it" is doing prealgebra and the so-called language instinct is also a mathematics instinct.

Environmentalism as Model

When I was growing up the concepts of "the environment" and "environmentalism" did not exist. Of course we had problems that are addressed today by environmentalists. There was pollution of rivers and soil erosion and deforestation and even an accumulation of hothouse gasses in the atmosphere. But these problems were small enough, and changed slowly enough, to be handled in piecemeal fashion. There were professionals for each of them. There was nobody whose job and professional competence was to deal with the whole of which they are parts, until events such as the 1962 publication of Rachel Carson's *Silent Spring* precipitated a movement that would soon give rise to a truly fundamental idea: thinking in a holistic way about everything that affects the waters, the airs, and the lands of our planet.

I believe that the time has come for an educational concept similar in its holistic nature to "the environment," an entity one might call the Mathetic Environment - everything that affects learning in all its forms. I use the words "mathetic" and "learning" almost interchangeably. By "mathetic," I mean related to learning; the Greek stem "math-" originally referred to learning as in "polymath," which refers to a person with multiple learnings, not to a mathematician. Until recently little harm was done by the circumstance

that there are specialists on innumerable aspects of learning but nobody whose job and competence are concerned with this whole. As a first step towards explaining what I mean by this, I extend the Roman-Arabic thought experiment as a parable to pinpoint one of the consequences of fragmentation in the field of learning.

Elementary school math specialists are hard at work on very specific problems like undoing misconceptions such as "since LX is ten more than L, LIX should be ten more than LI." The idea of abandoning Roman numerals had occurred to one of them but was immediately dropped: the children would not be able to manage the higher grades. The NCTM had tried to issue standards for all grades based on the new system. But the universities complained and the parents screamed that the kids aren't learning the "real math" they had learned - moreover, all the money they had spent on buying software to prepare the toddlers for school would be wasted. And, besides, the NSF reviewers... well enough, the point is char.

I use a parable to make my point because it is easier to get a consensus that Arabic arithmetic is better for society than it is to get a consensus on any of the many real examples of change that are impeded by similarly antiscientific reasons. The main point I want to make is that significant change in what children learn may require thinking outside the fragmented boxes of the education system. It is a challenge to the learning sciences to find ways to do so.

Love and Fear in the Mathetic Environment

The rapid formation of environmentalism cannot be understood in affectively neutral terms. Carson and others made people aware of frightening dangers; I suspect we may have been lucky that this happened at a time when so many people were in love with such ideas as "holistic" and "nature." In the case of the mathetic environment, my belief in the possibility of a tipping point is fueled by the existence of causes for fear and potentials for love. I begin with the negative side and mention two reasons for alarm.

The biggest effect of computers on school comes from computers out of school and is deeply negative. Every child can see that the school's ways of using computers are not how the increasingly digital society does things. To my mind there is no doubt, and certainly it is a plausible conjecture for scientists of learning to examine, that the growing disaffection with school comes largely from awareness of this gap. I see disaffection most directly in statistics that show a yearly increase in the number of high school students in the United States who declare that what they learn in school is irrelevant to their lives. I see it also in the growing epidemic of "learning disabilities," a name that is belied by noting how often they somehow do not impede the learning of complex computer games. I believe that this growing trend cannot be reversed by "better" teaching of what the children correctly see as obsolete knowledge; it can only be reversed by changing what is learned and taught.

I mention my second promised reason for alarm before hinting at the principles that might guide the selection of new content. There are a billion children in the world who have access to global information, including a direct view on TV screens of a better life than theirs, but do not have the learning opportunities that might enable them to be part of that life. I am not sure whether it is up to the learning sciences to understand more deeply how this deprivation of learning contributes to hatred, violence, and instability. Perhaps this is in any case quite obvious. What I do know for sure is that it has to be on the agenda of the learning sciences to find ways to bring modern learning to these billion people and to do it fast.

It seems obvious to me that the scale of these two alarming situations is such that the means to deal with them will have to be based on mobilizing the same powerful technologies that have caused them. But of course technology alone will not do it. Finding the right ways to use technology is a serious - perhaps the most urgent - challenge facing the learning sciences. But it is easy to find a starting point.

Across the globe there is a love affair between children and the digital technologies. They love the computers, they love the phones, they love the game machines, and - most relevantly here - their love translates into a willingness to do a prodigious quantity of learning. The idea that this love might be mobilized in the service of the goals of educators has escaped no one. Unfortunately, it is so tempting that great energy and money has been poured into doing it in superficial and self-defeating ways - such as trying to trick children into learning what they have rejected by embedding it in a game. Nobody is fooled. The goal should not be to sugar coat the math they hate but offer them a math they can love.

The Mathetic Cupid's Arrow

I became a mathematician by falling in love with mathematics. The passage in my own writing that has been most often quoted and reprinted is a description in my book *Mind-storms* of how I fell in love with mathematics by first becoming attached to a "transitional object" - as it happens,, mechanisms involving wheels and gears - which was more meaningful for a young child. The idea which is critical to my present theme has been made more so by two very recent shifts in my thinking.

Until recently, I used the language of "falling in love" quite loosely, as meaning metaphorically nothing more than an intensive form of "like." The first shift consists of exploring the possibility of a more literal meaning and so opening a whole branch of study for the learning sciences. Recent brain studies offer hope of identifying a neural basis for falling in love and for monogamous mating. I have suggested ways in which these studies could be modified to explore the possibility that falling in love with an intellectual topic or an idea could be neuro-logically as well as metaphorically related to falling in love with a person and that the kind of devotion to a subject that many people develop could be related to the phenomenon of monogamous attachment to a person. If these conjectures have any truth, what we do at school in the name of motivation would be exactly wrong. Teachers try to make every child enjoy the mathematics we have chosen to teach. But when a person falls in love with another this is very different from falling in love with people in general - indeed, almost the very opposite.

My second shift, related to this last remark, modifies what I have long taken as my guiding principle for research in mathematics education: *Instead of making children learn the math they hate let's make a mathematics they will love.* The idea that one could make a mathematics that all children would love now makes me uncomfortable. Instead, I would now set as my guiding goal to give children the means to find unique ways to create a personal mathematics of their own to love.

This might seem like a copout: surely society decides what its citizens should know; and the job of the learning sciences is to facilitate their knowing it. But recall the parable of the shift from Roman to Arabic arithmetic. What "society wants" is not for people to have the ability to manipulate particular symbols whether they are C, X, V, and I or 3, 2, 1, and 0; society wants people to be able to think about numbers and to use them to think about other things. And this is equally true of all mathematics and indeed of all areas of thinking.

There is, of course, a more serious objection: even if the goal is desirable surely it is not attainable; making a mathematics means building a formal system and this is something that only a few highly educated adults have ever done. My answer approaches the crux of my essay: the objection *was* valid until quite recently, but we see it becoming unraveled by looking at another successful new science - computer science. Today, every one of several million programmers is engaged, although not necessarily in a self-conscious or even useful way, in building formal systems. The deep meaning for children of giving them the ability to program computers is to provide the tools for them to do the same. In my vision, whatever they know intuitively, implicitly, innately, and whatever they care about passionately, could be expressed in the creation of a formal system that is entirely theirs. The deep meaning for the learning sciences of allowing children from the earliest ages to learn an appropriate form of programming is to create for the first time the possibility of freely exploring the infinitely open-ended variety that forms of knowledge and their learning can take.

Making Drugs, Genes, Elements, and Knowledge

Before history, we were scouring nature to find plants that would heal. Later we extracted the compounds responsible. Still later we synthesized them. But in this we were still using what existed independently of us. Recently we are beginning to do something very different: develop substances that never existed in order to produce an effect we understand. One can see a similar progression from a type of breeding that acts indirectly on genes that exist, to modifying them directly, to making them. Alchemists tried in vain to turn one element into another. Today we do it routinely and even make elements that never existed.

Educators in the past have taught knowledge structures that existed independently of them. I have been suggesting that the learning sciences might follow the chemists, the biotechnologists, and the physicists by

purposefully making *with a mathetic intent* knowledge structures that never existed. Curriculum designers may protest they have always been doing this. The distinction might be blurry, but I believe that it is worth making and worth studying how to make it better. I do not think one should describe the shift from Roman to Arabic numerals as "making a new curriculum" for arithmetic. That kind of shift is something very different. I began this afterword by suggesting that it might be the exercise ground for seeking the mathetic fundamental. I close by suggesting that finding the conceptual framework for that shift might be the critical fundamental problem our science needs. What does it mean to have mathetically different forms for knowledge? What makes some more learnable? What makes some more lovable?

Footnotes

- i. Throughout this essay I use the word "mathematics" as a stand in for all disciplines. I use the word "math" to refer to the largely obsolete stuff they teach in schools.
2. A theoretical framework for this kind of relationship between mathematical structures will be the subject of a projected paper by Uri Wilensky and myself