Stimulating different intelligences in a congruence context

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Abstract

The paper deals with didactical scenarios on the notion of *geometric congruence* based on previous successful experience with using a specific model T which reflects the relations among structures, activities and intelligences (in the sense of Howard Gardner). The underlying idea is that different representations of the same object of study (Figure 1) enable the learner to use his/her strong intelligences, and to enhance and develop the rest in the intelligence spectrum.

Figure 1. Various representations of geometric congruence

We present a series of problems and the response of students of different age. The learning process is in harmony with the fundamental ideas of the constructionism – it involves computer modeling of artefacts created in the best tradition of the Bulgarian folk art – wood-carved ceilings (Figure 2), painted ceramics, embroidery, crocheting, etc., to be presented and shared within a larger audience.
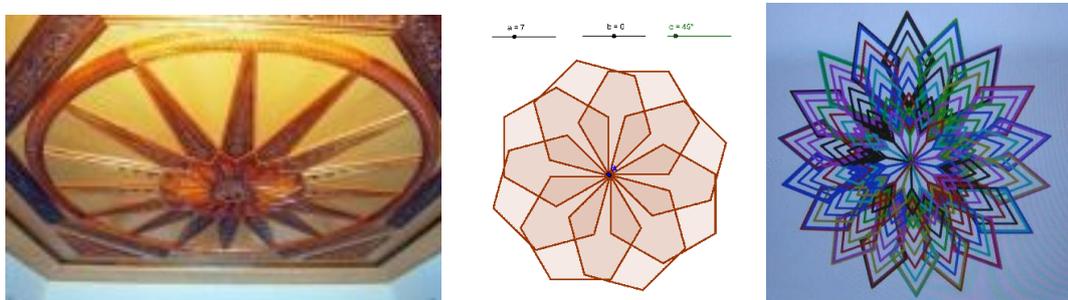

Figure 2. A wood-carved ceiling and models in different computer environments

Our experience with students and teachers alike suggests that harnessing the strong intelligences of the learners enhances the understanding and appreciation component of the learning process.

Keywords

Constructionism, geometric congruence, art, intelligence

Introduction

The symmetry and the asymmetry co-exist in unity both in nature and art. A creative process involves specific decisions about the symmetry-asymmetry ratio of the object being created, and this very ratio is a crucial aspect when evaluating its esthetics. The design and the creation of objects sharing the features of fantasy and nature is often based on geometric transformations such as *congruences* (translations, rotations, reflections and compositions of those). Thus it seems natural to encourage learning this geometry topic by integrating it with the design and reconstruction of artistic artefacts. With this idea in mind we have developed didactic scenarios on geometric congruences in the frames of the *InnoMathEd* European project [1]. The design and the implementation of these scenarios is just an element of a more ambitious goal - we expect our students to look for manifestations of geometric congruences, discover them and use them in various activities, and thus – to be able to find patterns and relationships deepening their knowledge and understanding of the surrounding world.

The congruence scenarios are based on previous successful experience with using a specific model T [2] which reflects the relations among structures, activities and intelligences (in the sense of Howard Gardner [3]) - Figure 3.
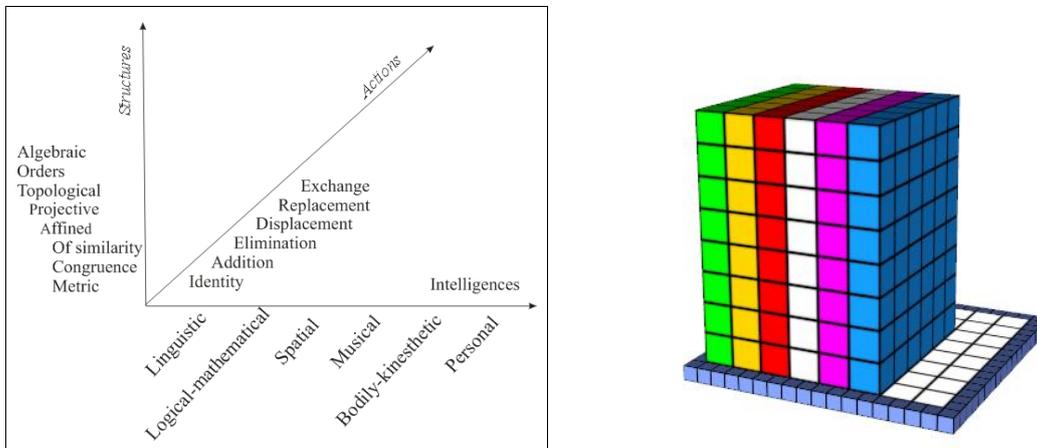

Figure 3. The structure–activity model T of the intelligences

The congruences

The notion of *congruence* can be introduced by various methods and activities involving different intelligences (a subset of the T model).
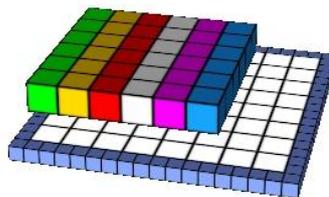

Figure 4. The congruence layer

The students:

- observe objects from nature, architecture, popular customs, science and discover (recognize) congruencies
- create congruent objects by means of various ideas and tools (e.g. by folding a sheet of paper, turning a slide or a piece of glass with a drawing on it, by means of an appropriate software, etc.)
- transform figures in order to get a specific one
- reach the level of free use of geometric transformation as a means for achieving the goal of a more complex project

The first tasks for the students are to discover and describe the harmony based on congruences in different sports and folk dances (Figure 5). Then they are expected to perform symmetrical movements with arms and legs. And finally – to present (just with their hands) geometric figures such as triangle, trapezoid, etc.
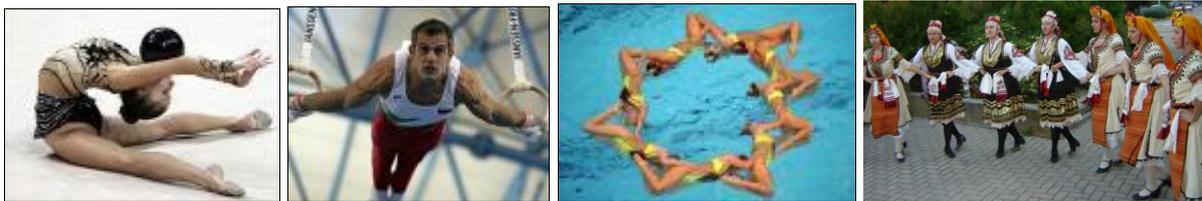

Figure 5. Recognizing congruences in sports and dances

Next the students can use various computer environments for exploring congruences [4-7]. They can draw by "free hand", work with specific tools in graphic editors, grids in 2D and 3D, dynamic constructions, and of course - programming in Logo (Figure 6).
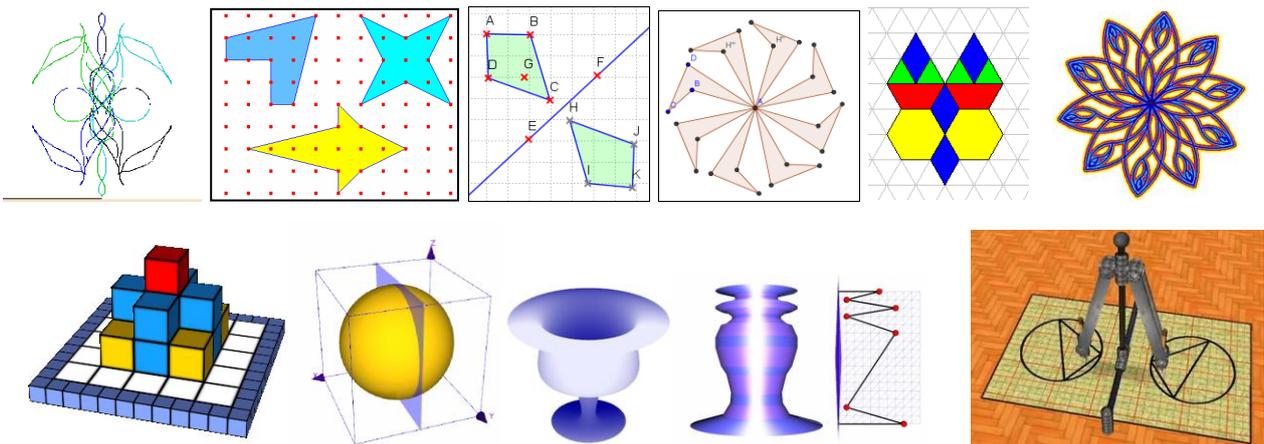

Figure 6. Different computer environments for exploring congruences

In order to make the right choice for a specific project it is important for the students to realize that each environment has its advantages and disadvantages.

For those who are fond of music, the symmetry could be demonstrated in the context of famous music fragments: (Figure 7). Here is how a 16-year old student with a high motivation for science reacted to this musical illustration of symmetry during a lecture on music and mathematics:

> When Prof. Noam Elkies played the traditional Paganini theme in reversed it turned into variation 18 of Rahmaninoff's Rapsody on the Theme of Paganini. For anyone with at least an ounce of knowledge in classical music, this had to have been an epiphany moment...

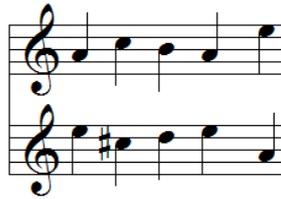Thus the symmetry could not only be seen but could also *heard*.

Figure 7. Part of 24th capriccio theme by Paganini and the 18th variation theme by Rachmaninoff

A good project for the musically inclined is to find the musical interpretation of the geometric congruences [8].– Figure 8.
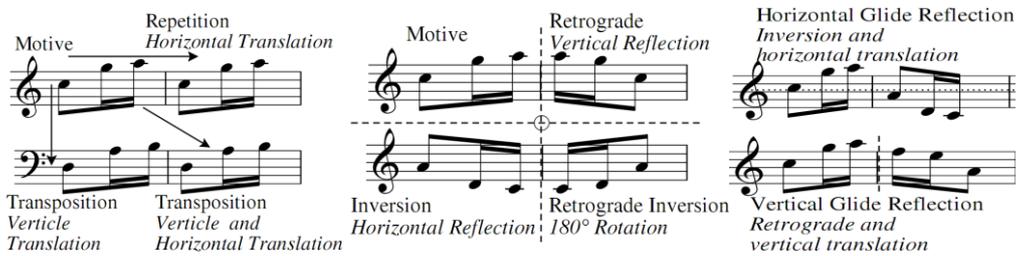

Figure 8: All congruence transformations in musical space

Congruences could be found in various aspects of a music piece when considered as a sequence of clear-cut units. The repetition (translation) and the symmetry in a melodic structure are easily recognized in its formal description (by letters or digits) (e. g. AABBCCDD, ABA, 11112233, 11122111) where the same letter (digit) would mean that the corresponding units have the same melodic structure possibly modified by transposition (keeping the intervals). The same idea is used for describing the harmonic structure, e.g. the string TTSDDSTT reflects the harmony of 8 bars being based on the tonic (T), subdominant (S) and dominant (D) chords.

Analyzing the structure of a set of music pieces in terms of congruences can be very helpful within a project for writing algorithmic music [9]. Similarly, it is a good challenge for the students to recognize congruences in the natural language and the literature works (e.g. to search and check for palindromes, to analyze the structure of a drama, of poetic forms such as sonnets, haiku, etc.). Then they could describe formally these structures, and finally – to get acquainted with the standard description, e.g. in the case of drama – with *Freytags pyramid* (Figure 9) [10].
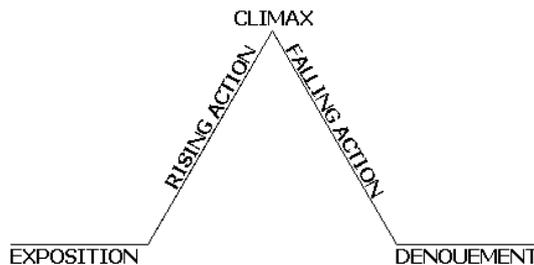

Figure 9. The structure of a drama as presented by Freytag [10]

Especially interesting in mathematics context are problems with the following operations: adding, removing, replacing, and exchanging [11 - 14].

What follows are fragments of our experience with design and reconstruction scenarios involving congruences. We present a series of problems and the response of learners of different age.

Congruences and design

Problem 1. Study the tessellation below and create your own tessellation design.
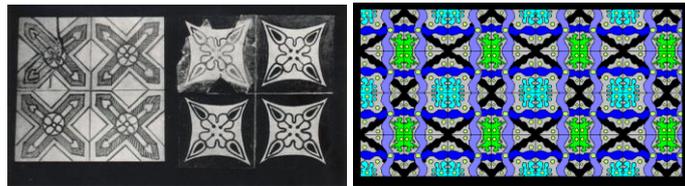

Figure 10. Painted ceramics from Veliki Preslav and a modern tessellation

After observing the above samples of tiles the students are expected to realize that since the tile has the form of a square and decoration with 4 axes of symmetry the options for different tessellations based on these tiles are very limited (if the tiles have to have common vertices).
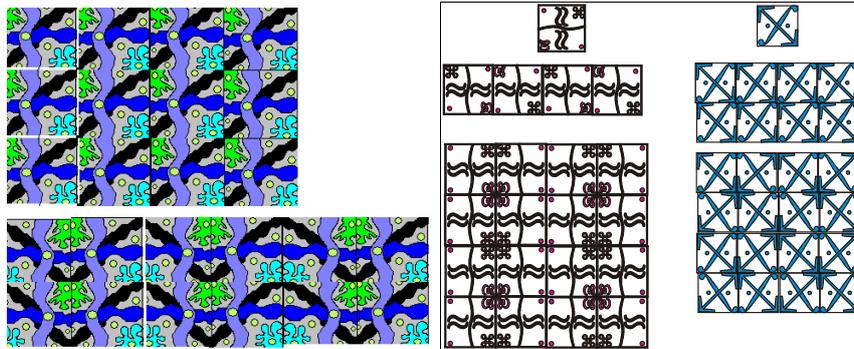

Figure 11. Tessellations with a ready-made and with learners' own tile-model

This project was used in educating pre-service teachers who were given the opportunity to use software of their choice. The computer models in Figure 10 and Figure 11 were made with Paint and CorelDraw. The strength of using a programming language (Logo in our case) was obvious – precision of the tiling, using procedures with parameters for the size of the tile, the number of the rows and columns and turning it in a dialog with the user made the programming efforts worthwhile.

Problem 2. Make a model of a ceramic plate in the style of "Troyan drop" [15]. (the first two in Figure 12).

Figure 12. Samples of hand made ceramic plates and computer models in the same style

When painting in a *free hand* style the students would start with the axes. All of them used an even number of axes and then continued the painting by coupling the motives. The zoom mode was used to improve the precision of the work.

Problem 3. Help the archeologists to reconstruct the artefacts (Figure 13) by creating computer models.

Figure 13. Models of ancient artefacts to be reconstructed

One of the shortcomings of using Paint is that only the rotation at angle multiple of $90°$ is possible. If the students choose dynamic software, e.g. GeoGebra, they can insert twice the picture and then rotate the first copy around a center at an appropriate angle. The first figure is made transparent and then the two figures are rotated until they fit (Figure 14).
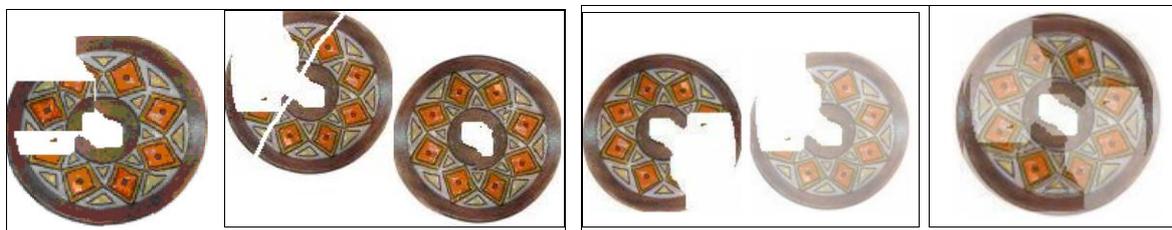

Figure 14. Reconstructions by means of Paint and GeoGebra

The idea could be extended in the case of rotational artefacts.

Problem 4. Create computer models of the artefatcs in Figure 15.

Figure 15. Modern and ancient artefacts based on rotation

Some of the students decided to use a pattern to be copied and translated appropriately (Figure 16). The difficulties when passing from 2D to 3D were discussed with the teacher.

Figure 16. The students' design

Problem 5. Make a model of a wood-carved ceiling as the ones in Figure 17.
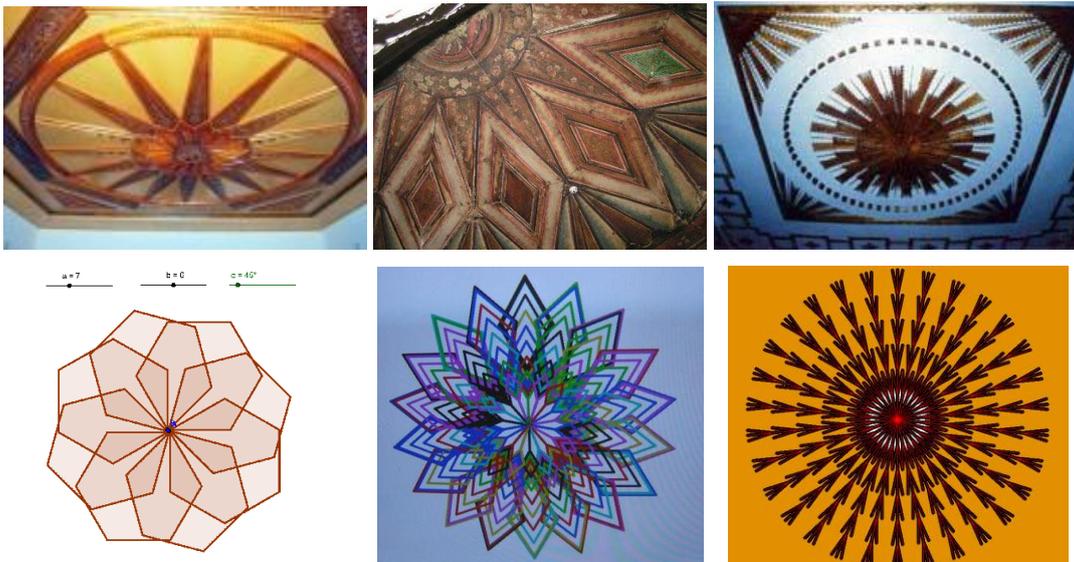

Figure 17. Woodcared seiling from Triavna and Plovdiv and some computer models

In the case of the last model in Figure 17 the students started with a procedure drawing a Y-shaped element, then added parameters for the size and the angle, made a branch with a varying number of Y-shaped elements and rotated it around the starting point giving rise to a whole class of ceiling models.

Another rich idea for design projects is the Bulgarian embroidery and crocheting.

Problem 6. Create a computer motive for Bulgarian embroidery (Figure 18-1, 2).
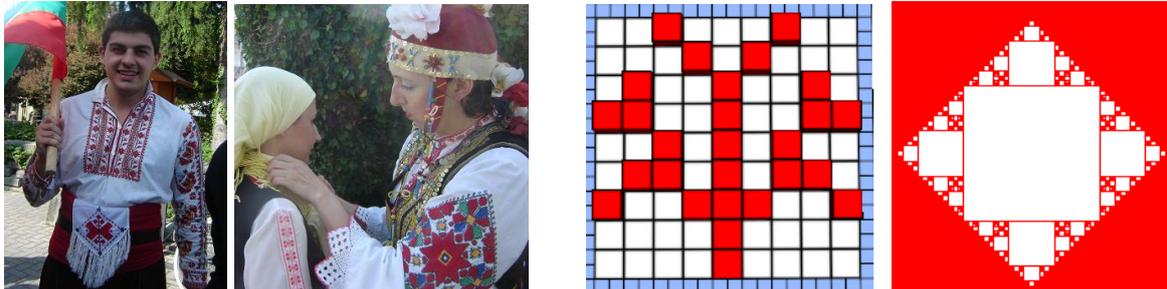

Figure 18. Bulgarian folk dancers and computer models inspired by the embroidery of the costumes

Here the students are expected to assess the specifics of the software and to choose the most convenient one for creating the basic motive and then to decide what geometric transformation to apply so as to get the desired composition.

The project for creating crochet models involves precision and option to play with various parameters, thus the choice of Logo is natural:

Problem 7. Create a design of a lace napkin for serving tea or coffee (Figure 19-1).

Figure 19. Masterpieces of older and younger Bulgarian women

It is interesting to note that this project is usually offered at the beginning of a Logo course for graduates in different fields who want to qualify for teachers in IT. As a rule they get excited of the "no threshold, no ceiling" motto of Logo and enjoy playing with parameters to produce various rosette models, and further – to create an interactive program. Of course, the ceiling could be only your own imagination. What if the crochet model contains a unique element which happens to be symmetric (Figure 20-1)…This gives rise to the next project.

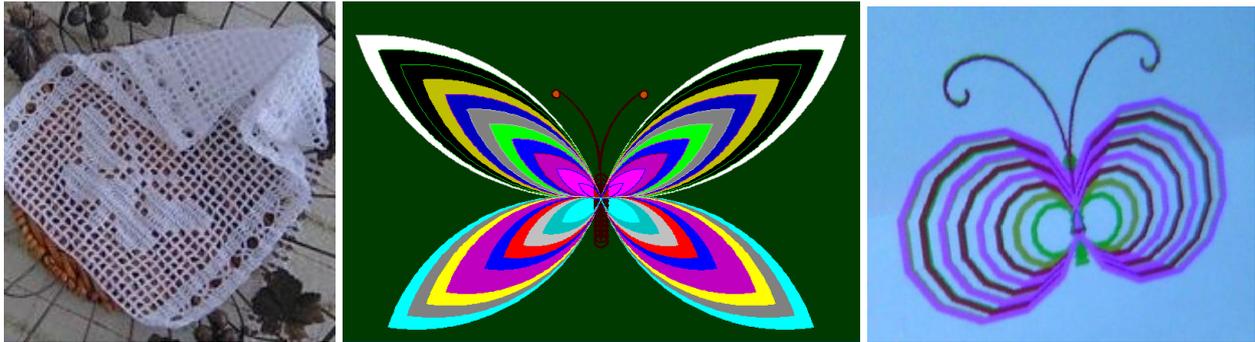**Problem 8.** Create a stylized model of a butterfly:

Figure 20. Crochet and Logo models of a butterfly

Coming back to our main object of study, the symmetry, let us recall what the great mathematician Hermann Weyl thinks of it: *Symmetry is one of the ideas by which man through the ages has tried to comprehend and create order, beauty, and perfection.*

Is there an innate link between symmetry and beauty? Could symmetrical composition be an intricate part of defining beauty? Plastic surgery advertisements often read: *The symmetry is beauty.* At the same time a recent study by Harrison [16] shows that although we commonly perceive faces as symmetric, in fact there are asymmetries in them.

Maybe this interplay between symmetry and asymmetry in search of beauty was what inspired an in-service teacher (working in a school for fashion design) to create a pattern which was originally symmetric and then became asymmetric after she added a decoration (Figure 21)**:**

Figure 21. Creating balance of symmetry and asymmetry

Discussion

Many authors have been looking for various means to support the understanding of knowledge, including the notion of *congruence*. The T model provides options for seeking completeness with respect to the intelligences and the actions used. There are examples in the literature for the use of linguistic, spatial, and musical intelligence but we haven't come across a goal-oriented reference to the bodily-kinesthetic intelligence in this aspect. Often the teachers create situations for recognizing and building structures involving congruences and addition. Such situations could be visualized by a subset of the T-model (Figure 22-1).
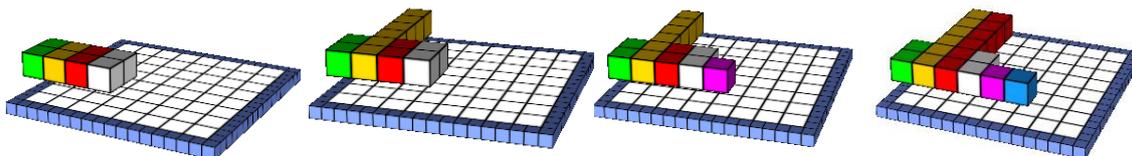
Figure 22. Visualisng systems of problems on congruence involving various intelligences and actions

Ideas for including the actions *adding, elimination, displacement, replacement* and *exchange* in systems of mathematical problems on congruences are presented in [11-14] and could be visualized as in Figure 22-2. If the rest of the itelligences are involved the completeness of the congruence layer (Figure 4) would be achieved.

The problems involving the bodily-kinesthetic intelligence could be presented as in Figure 22-3. When organizing the learning process in harmony with the constructionists ideas it is important to emphasize on the sharing. It is related to thinking on the thinking, to the estimating one's own actions, to understanding and estimating the actions of the other members of the team – all being components of the personal intelligence (Figure 22-4).

The T model could be used also at a problem level. When preparing a problem to be offered to the students the teacher has expectations about the possible solutions and mistakes the students would make. Based on the various possible solutions the teacher can fix the actions being used by his/her students. Thus some *gaps* will appear which could be filled by new problems.

Conclusions

Our experience with learners of different age (pupils, pre-service and in-service teachers alike) suggests that harnessing the strong intelligences of the learners enhances the understanding component of the learning process.

As emphasised by Clayson [17] the visual exploration with models helped the learners increase their visual vocabulary by forcing it to describe (sometimes) unexpected shapes.

Furthermore, assisting the learners in their choice of a computer environment appropriate for a specific project goal is a crucial factor for the students to experience satisfaction, joy, self-confidence and motivation for further activities, and even to reach thoughts impossible for them before that.

In conclusion, we felt very encouraged to hear what one of the teachers we have recently been working with wrote after a Logo course: *It is not possible to "feel" informatics, music, art, poetry, even dancing, not to forget the dreams, if they are in isolation.*

Acknowledgemnts

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