

# Eight years of journey with Logo leading to the Eiffel tower mathematical project

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## Abstract

Eight years ago, our school in Mexico was introduced to Logo as part of the government-sponsored Teaching Mathematics with Technology (EMAT) program. Since then we have increasingly become interested in developing long-term, interesting, constructionist (Harel & Papert, 1991) projects for our students, particularly in the last 5 years. The idea is that through these projects students become engaged and motivated, while they learn – in a fun and meaningful way – many mathematical topics in the official syllabus, but also have early access to other “powerful ideas” (Papert, 1980) such as “advanced” mathematical concepts that are usually not even considered for children of the age-groups we work with (12-14 years-old), as is the case of trigonometry. At Eurologo 2007, we reported our first long-term “Painless trigonometry” project (Jiménez-Molotla et al., 2007). Our latest project, the “Paris project”, inspired by the hosting city for Constructionism 2010, evolved from that previous trigonometry project, and had as aim the construction of the Eiffel tower (Fig. 1). Such construction (Fig. 2), done in 3D MSWLogo, has required an understanding and use of trigonometric ideas, such as the Pythagorean theorem, and mathematical analyses using various tools (including Google Sketchup and Scratch) of geometrical objects such as pyramids and prisms, which form the building blocks of the Eiffel tower representation. As in our previous projects, the children have been highly motivated and engaged, and their creativity and genius has been awakened.



Figure 1. The aim of our “Paris project”: the Eiffel tower

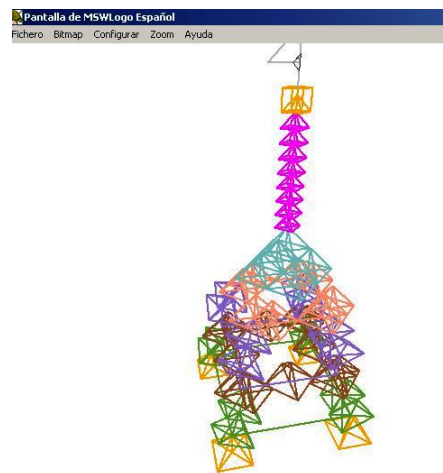


Figure 2. A student's representation of the Eiffel tower in 3D Logo

## Keywords (style: Keywords)

Pythagorean theorem; pyramids; 3D geometry; school project; Logo; Google SketchUp

## Background: The evolution of the trigonometry projects

In 2001-2002, our junior secondary schools (children aged 12 to 15 yrs-old) in Mexico were introduced to the Teaching Mathematics with Technology (EMAT) government program, which promotes a constructivist use of open tools (where the user can be in control and have power of deciding how to use the software) such as Spreadsheets (Excel), Dynamic Geometry (Cabri-Géomètre), and Logo (MSWLogo). As we became more proficient in the use of the tools, particularly of Logo, we developed our own activities and projects. Thus, in the past 5 years, we have been working in developing interesting constructionist long-term mathematical projects for our students with an integral use of technological tools like EMAT's Logo, Cabri and Excel, but also with other creative and expressive software. These projects give students an avenue for learning many mathematical topics in the official syllabus, in a fun and meaningful way, while they also give them early access to "powerful ideas" and "advanced" mathematical concepts, such as trigonometry, which is usually not even considered for our Grade 1 and 2 children (12-14 years-old).

Trigonometry is a topic that is traditionally difficult to teach and learn. But computational tools such as Logo, allow early access to this important mathematical area through fun constructions. On the one hand, through these projects, young students can become familiarized with this topic, so that by the time they get to Grade 3 and have to formally learn some trigonometry, they will have experiences and useful intuitive ideas (diSessa, 2000) to build upon. On the other hand, through trigonometry, we can cover other mathematical topics in the curriculum such as: addition, subtraction, multiplication and division; powers and square root of whole and rational numbers; algebra (including constants, variables and polynomials); and geometry.

At Eurologo 2007, we reported on our first venture in this area (Jiménez-Molotla et al., 2007). We described how, in the academic year 2005-06, we took up the challenge to create a technology-based approach for the learning of trigonometry: the "Painless Trigonometry" long-term school project. In that project, we introduced our young students (12-14 yrs-old) to the Pythagorean theorem, basic trigonometry concepts and functions, and their applications using explorations and constructive activities with Cabri, Excel and Logo. This was a project that we carried in all our groups in grades 1 and 2 in two schools (approximately 250 students) for two academic years 2005-2006 and 2006-2007 (different students each year). Students thoroughly enjoyed the activities and gained interest in mathematics. They also developed problem-solving and collaborative skills. Furthermore, in written tests after the project, the students showed an understanding of the "advanced" trigonometry concepts, as well as of other algebraic ideas.

More recently, in Jiménez-Molotla et al. (2009), we reported how the painless trigonometry project had evolved in the academic year 2007-2008, into a project for constructing 3D pyramids. The new project was triggered by a question, from a student of the 1st grade (a group that was being introduced to Logo): "Is it possible to work in four dimensions in Logo?" This gave rise to the idea of a new school project for working in three dimensions, that was named for fun: "In search of the fourth dimension, while in three". We found a curricular topic for junior secondary grades 1 and 2 that could be worked in three dimensions, and that also gave access to the non-curricular theme of trigonometry: the pyramid. We started playing with paper-and-pencil in a geometry game to draw triangles and squares and whatever else was needed for a pyramid. We then transferred that activity to doing it with dynamic geometry (in Cabri) and used an Excel spreadsheet to help us in computing areas and perimeters. In the end, the children programmed pyramids in Logo, and some of them even achieved animations so the pyramids would rotate.

## The new project: The Eiffel Tower

When at Eurologo 2007 in Bratislava we heard that the next conference would be held in Paris, immediately we thought that a theme for a new project could be the Eiffel Tower. But it wasn't until 2009 that the project could take place, since we needed to find ways for our students to have the necessary tools to fulfil what we had in mind. For example, students need enough competency in Logo programming, they need to develop an understanding of basic geometry (angles, triangles, polygons, circles, etc), they need to be able to solve arithmetic and algebraic problems, and, of course, they need to work in three dimensions. But the previous projects, particularly the one involving constructions of pyramids, showed us the path for the fulfilment of this new venture.

In this academic year 2009-2010, we have been working intensely for some 5 months on the project (as in previous years, we have worked with approximately 250 students of grades 1 and 2 groups in two schools), having only one 50-min. session per week in the computer room; the results have surpassed our expectations.

We like to think that this project considers the compulsory syllabus that we have to follow in our schools, as we are required to cover certain topics; and it does so, by all the mathematics (as listed above) that it involves. It also tries to use technology, as is recommended in the new official programs (which recommend explicitly the use of the EMAT tools) but it tries to do so in a more creative way than is usually seen in other schools. Our approach is constructionist, since students themselves build the project; we only give ideas or questions, but students themselves pursue them. For example, we may suggest that they build a procedure for constructing a general right triangle with equal catheti; students analyse the problem, share their solutions, and collectively pick correct procedures. This is how we work in general, and "children learn that the teacher too is a learner" (Papert, 1980, p.114). This contrasts with happens in traditional classrooms; in this sense we completely relate to what was also said by Papert (1980, p.115):

In traditional schoolrooms, teachers do try to work collaboratively with children, but usually the material itself does not spontaneously generate research problems. Can an adult and a child genuinely collaborate ... A very important feature of work with computers is that the teacher and the learner can be engaged in a real intellectual collaboration; together they can try to get the computer to do this or that and understand what it actually does. New situations that neither teacher nor learner has seen before come up frequently and so the teacher does not have to pretend not to know. Sharing the problem and the experience of solving it allows a child to learn from an adult not "by doing what teacher says" but "by doing what teacher does."

## The development of the "Paris project"

We tackled the Eiffel tower project (which we nicknamed the "Paris project") using as a basis pyramids (which in itself is a challenging project, as we had seen the previous year). The project began with paper-and-pencil work (Fig. 3), using a geometry set, to think and get a clearer idea of how to tackle the design of the project. We also let students reflect on how to construct prisms and pyramids – we used the idea of prisms as building-blocks for building pyramids (we also studied volume formulas on the way). Google SketchUp was a good tool for visualizing the prisms and pyramids (Fig. 4).

Simultaneously, and since we have no resistance in taking advantage of new tools that come along, we started a blog (see Fig. 5) in Wordpress, with which we interact with our students and which their parents can see. In this blog we have been posting the progress of the classroom activities and of our project (with a table similar to the one shown in Table 1), as well as tips and tools that students can use. Through this blog students leave comments and participations, that help and enrich communication in the classroom, as well as with parents and authorities. In this way we are using social networking as educational tools, and the results have been very good.

Table 1 shows the sequence of activities that was followed with Logo (and Scratch) – a very similar table was posted on the blog. We introduced students to Logo and Scratch (as well as Cabri) only in October 2009 and in just a few months their progress was outstanding (those students who had participated in the previous year’s Pyramids in 3D project had moved on to higher grades or had new teachers, so all the students we had this year were new to Logo, the other computational tools and the activities). We began by exploring quadrilaterals with paper-and-pencil and geometry sets, with Cabri (Fig. 6), and then with Logo as well as Scratch. This is an easy way to learn Logo and we let our students play with their constructions. For students, things are more meaningful if they can play with them or if they have a challenge. For example, when constructing squares, we challenged them to create a staircase but this soon became something else when they played with their Logo procedures (Fig. 7). Some people have told us to stop children from playing, but the answer is: if it’s play to them, let them play.



Figure 3. Students’ paper-and-pencil work.

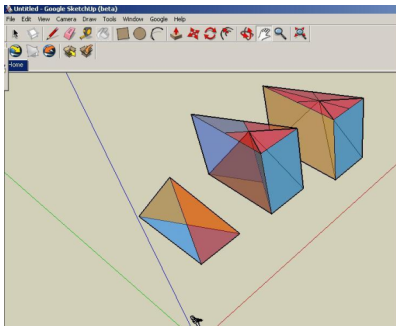


Figure 4. Prisms and pyramid constructions with Google Sketchup, as a visualization aid.



Figure 5. Image of our blog where we posted the “Paris project”.

Table 1. Summary of activities in the “Paris Project”:

Aim: construction of the Eiffel tower		
Activities	Logo	Scratch
1. Beginning with the construction of squares...	Getting to know Logo (Fig. 6)	Getting to know Scratch
2. ... students will construct regular polygons with different number of sides	Regular polygons	Regular polygons
3. Through the construction of an equilateral triangle, students reflect on the inner, outer and supplementary angles of a triangle, and see that the sum of the	Equilateral triangle	Triangle

inner angles of any triangle equals $180^\circ$		
4. Squares and triangles...	...for building a house	
5. Squares, triangles and rectangles...	...for building castles	
6. Welcome to the third dimension	Building a cube (Fig. 8)	
7. First encounter with a very important tools: The Pythagorean theorem (with right triangles of $45^\circ$ angles)	Half a cube: Trace the diagonals across two faces of a cube to split the cube in half.	
8. Construction of a rectangular prism and of a cube	Build a car using a rectangular prism and a cube (Fig. 9)	
9. Prisms and pyramids	Build a house using a rectangular prism and a triangular prism (Fig. 10)	
10. Triangle and segment...	... for building a flag. Challenge: animate the flag so that it rotates on its axis (Fig. 11)	
11. Rectangle, equilateral triangle and segment (tower with flag)	Place a flag on top of a tower (Fig. 12)	
12. Animations	Programming for creating animations of squares and polygons	
13. The Pythagorean theorem, triangles, rectangles and squares	Build the base of a pyramid and its height (Fig. 14)	
14. The Pythagorean theorem and right triangles with equal legs and $45^\circ$ angles	Finish the pyramid using right triangles with two equal sides and $45^\circ$ angles	
15. Students build theorems	Theorems are helpful in building different types of triangles.	
16. Building pyramid stacks	Note: Students ran into problems when they tried to build pyramids on top of each other. They realized they needed to begin and end the procedures in the centre of the pyramids.	
17. Building a new pyramid	Students refine their pyramid procedures for using less sub-procedures and so that they are easier to combine	
18. The pillar of the tower	Construction of part of the Tower	
19. Each student decides how his Tower project will be finished, and they share and express their difficulties and progress	Students decide how to finish their project based on their own reflections. They share amongst themselves their progress, and parts of their procedures, to help each other.	

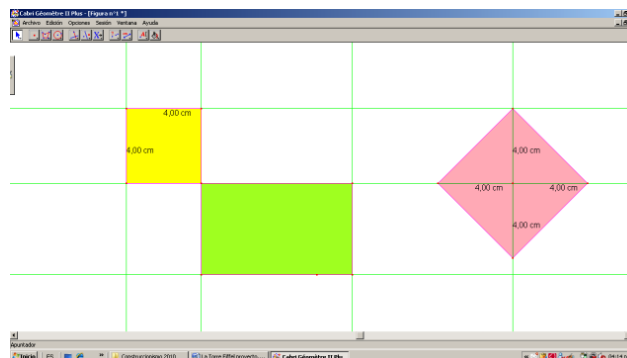


Figure 6. Cabri explorations with quadrilaterals.



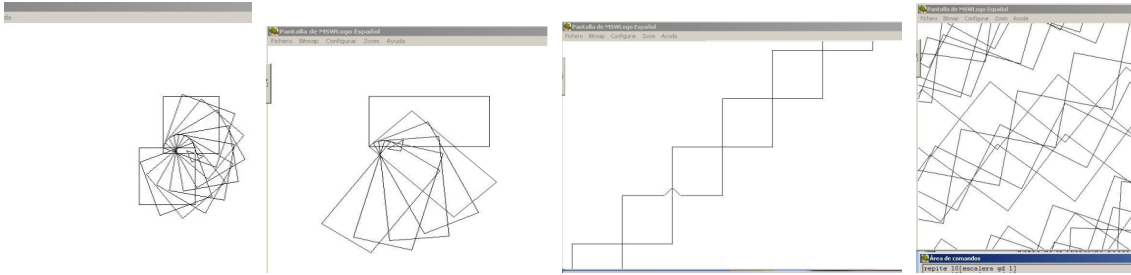


Figure 7. Logo play with quadrilaterals.

We continued with constructions of regular polygons with technology (activity 2 in Table 1) and without it. In the regular classroom we did paper-and-pencil analyses of the hexagon as a figure formed by 6 equilateral triangles; this was a way of introducing the importance of the triangle that will be very important for the project. We then would introduce the Pythagorean theorem and three-dimensional constructions. We began work in the third dimension with simpler exercises than the pyramid, like building cubes and prisms, which were first visualized in Google Sketchup, then constructed in Logo (Fig. 8). Students were introduced to 3D primitives (such as those for roll and pitch) during the activity to build a cube in Logo. In this activity they needed to reflect on how to combine squares and the movements and angles the turtle needs to do. Afterwards they continued building rectangular prisms.

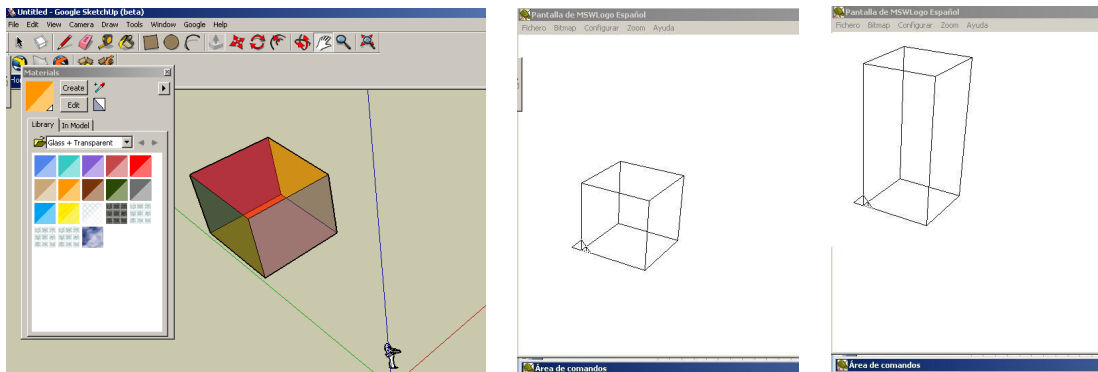
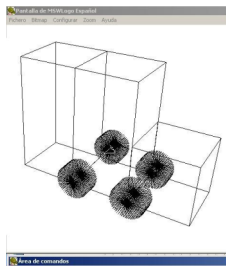


Figure 8. Visualization with Google SketchUp and constructions in Logo of 3D cubes and prisms.

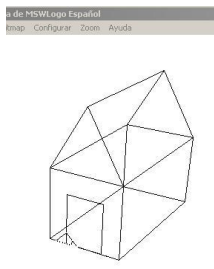
We challenged them to use those previous constructions (cube and prism) to build a car (Fig. 9) and a house (Fig. 10). The results were fabulous and students had fun while they learned and reflected.

```
to prism
repeat 4[square fd 100 downpitch 90 rectangle fd 200 downpitch 90]
end
```



```

to car
prism
downpitch 90
fd 100
downpitch -90
prism
downpitch 90
fd 100
downpitch -90
cube
end
    
```



```

to house
prism
fd 100
downpitch 90
fd 200
downpitch -90
rt 30
prismtri
fd 200
rightroll 60
downpitch 90
fd 100
rt 90
fd 75
rt 90
door
end

to prismtri
triangle
downpitch -90
rightroll 30
rectangle
rightroll 60
rectangle
downpitch -90
rt 30
fd 100
downpitch 90
rightroll 30
rectangle
end
    
```

Figure 9. A 3D Logo car (and procedure) built with cubes and prisms.

Figure 10. A 3D house (and procedure) using prisms.

We then moved on to the construction of a flag, which would be needed to be placed on top of the Eiffel tower representation. A challenge here was to animate the flag by having it rotate around its axis. Children love to create animations. When the flag was placed over a tower, children realized that they couldn't erase the entire screen between "frames" like they had done before, so they used the *penerase* primitive.

When we felt ready to move on to the construction of the pyramid, which would be the basis for the Eiffel tower representation, we used Google SketchUp for visualization and analysis purposes, and to understand how triangles form a pyramid (Fig. 13). In the classroom we also used paper-and-pencil and geometry set activities for understanding further the construction of prisms and pyramids (we also calculated areas and volumes). Students then began the construction of pyramids in Logo, beginning with the base and height (Fig. 14).

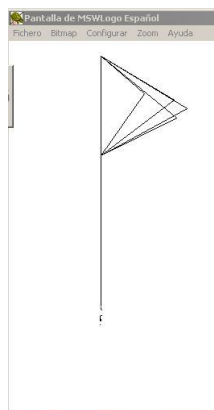


Figure 11. An animated rotating flag in 3D Logo.

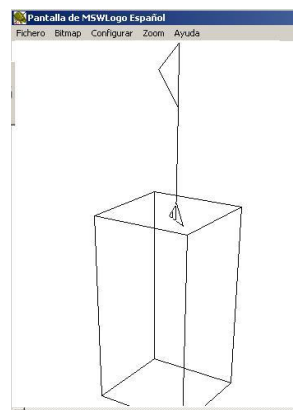


Figure 12. The rotating flag over a tower.

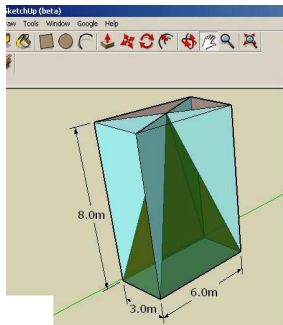
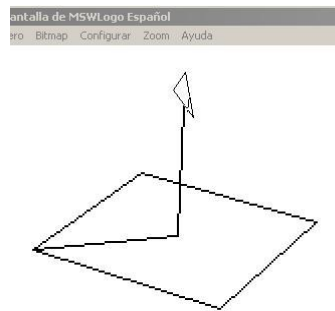


Figure 13. Visualization of the pyramid in Google SketchUp.



```
to pyramidbase
square
rt 45
fd sqrt (100*100+100*100)/2
downpitch -90
fd sqrt (100*100+100*100)/2
end
```

Figure 14. The base and height of the pyramid (with procedure) in Logo.

Finished pyramids would then be stacked to complete the Eiffel tower (see Fig. 2 on first page). A difficulty that was faced here was that for aligning the stacks of pyramids, the procedure for the pyramid needed to be changed so as to begin and end it in the centre of the pyramid.

Each student decided how to finish their own Eiffel tower; some children had real difficulties and challenges but they all shared their progress and those that were ahead shared parts of their procedures and ideas to help others.

Daniela, a 13 year-old student, was the first to finish the Eiffel tower representation (Fig. 2), and she added colours to make it nicer. She confessed to spending some 9 hours of work at home over three days to finish the project, showing the deep motivation that the project created in her. She wrote the following (translated from the original Spanish) on the project’s blog:

*“Logo is a computer software that really impressed me by its vast functions that one could modify by adding one’s own created commands. During my experience with Logo, I was deeply impacted in discovering the things that one can do alone with just a little bit of mathematics, interest and Logo. I also discovered another way to learn and develop my spatial and geometrical abilities. The truth is that for my final programming creation, there were many obstacles beforehand, I had difficulties with the 4 pillars [of the tower], and diverse things like colours, but after these “trials” that some consider mistakes, you have another perspective on how Logo functions. For me it was the best experience in mathematics and computer science.”*

These are some of Daniela’s main parts of her finished Eiffel tower procedure:

```
to pyramid
repeat 4[triangle rightroll 90]
downpitch 90
back theorem
left 45
square
right 45
forward theorem
downpitch -90
forward theorem
end
```

```
to triangle
forward theorem right 135 forward
theorem2
right 135 forward theorem
right 90
end

to theorem
output squareroot (50 * 50 + 50 * 50)
end

to theorem2
output squareroot (theorem * theorem +
theorem * theorem)
end
```



## Some results

As the project progressed, we observed many changes in our students. Their interest in mathematics was awakened, their understanding of regular mathematical school work improved, and they even developed a defensive attitude of their work in the project against outsiders who criticized it.

A student commented:

*"[It] was an adventure, in which one had to take risks and take different routes to reach the objective: the Eiffel tower, although I had a bit of difficulty with the bearings and I used as a compass, my previous knowledge; as a map, my classmates; and as a guide, my teacher, I explored different paths until finding the right one. A big adventure with Logo".*

There was also another student who was extremely aggressive at the beginning of the school year, and didn't do any work; even his father doesn't know what to do with him. This student became transformed during the computer work. Today he is a good boy in the computer room and is one of the first to finish the Logo activities (and though in the regular classroom he is still a bit undisciplined and doesn't pay attention, his overall work *has* improved). His classmates think he is a mathematical genius. As we have learned in our journey that began eight years ago, Logo helps us discover the genius in both the understood and the non-understood children.

## The obstacles

However, as in the past, we have continued to face many criticisms (and obstacles) from some peers, authorities and even parents, on our technology-based projects, because they don't understand what we are doing, despite the fact that the use of the EMAT program (though governmental support for teacher-training in that program has been stopped) is still explicitly recommended in the new mandatory curriculum. The work is not easy, as there is no real support for the use of digital technologies and related projects. An example of this was last year when we asked for support from the school authorities for attending a conference: the answer was no and that we shouldn't send any more papers in the future to conferences because there was no support (and yet we are doing it again).

Another example is when a parent filed a complaint with the school authorities saying that we only played with computers and children were not being taught mathematics (he actually wanted his child to do repetitions after repetitions of operations). He objected to work that was fun; for him mathematics had to be tedious.

The *kind of mathematics* foisted on children in schools is not meaningful, fun, or even very useful. This does not mean that an individual child cannot turn it into a valuable and enjoyable personal game. For some the game is scoring grades; for others it is outwitting the teacher and the system. ... for school math ... *despite* its intrinsic dullness, inventive children can find excitement and meaning in it. (Papert, 1980; pp. 61-62).

But mathematics *can* be interesting. Yet, working with technological tools like Logo carry great responsibility; it is not just a matter of doing a little bit and then abandoning it. That's why we like doing long-term projects. But most teachers object to doing this because they feel the curriculum and requirements are already too time-consuming that they leave no time to afford on these projects; they see only the parts and not the whole. We are reminded of Papert's words:

Conservatism in the world of education has become a self-perpetuating social phenomenon. [...] The computer revolution has scarcely begun, but is already breeding its own conservatism. ... [the] conservative social system appropriates and tries to neutralize a potentially revolutionary instrument. (Papert, 1980; p.37 and p. 45).

Papert (2006) argues that it makes little sense to use digital tools to serve a curriculum that was created without them, and he challenged us to reserve at least 10% of our time to think of what new mathematics and mathematical practices can emerge from the use of these technologies.

For these reasons we believe even more firmly that it is important to continue with our efforts to break the barriers. We believe that if students learn to use technology in a thoughtful way (moving away from a mechanized use and developing projects they feel as their own) they can become better learners, while new mathematical ideas may emerge, and the teacher can share and collaborate in their discovery processes rather than being the traditional presenter of knowledge. We are a new generation of students and teachers creating experimental environments, computational microworlds and classroom dynamics that are very different from traditional school practices. The catalyst for change are the technological tools, but it is in our power, as teachers, to actually make the change, and use the tools in innovative ways that change mathematical teaching and learning. The tools don't bring the benefits; it's the *use* we make of them that does.

## A further final comment on Logo

We would like to add here a comment that we consider important, with regards to Logo and to our project. When Jesús, the first author of this paper, went to Eurologo 2007 in Bratislava, he was very impressed by all the different versions and off-springs of Logo. Upon his return to Mexico, he felt temporarily that perhaps the version of Logo that was being used in our school projects (MSWLogo, which had been provided as part of the EMAT programme) was poor or outdated. But after a few days, he realised that it's not about the interface, and that the Logo language was in fact marvellous: Around that same time some officials from part of the Ministry of Education suggested that it was obsolete and outdated to not use other more modern tools, so he downloaded Scratch, with which we have also been working for a year. What we noticed by working with Scratch (which we also like) is that Logo is ageless and is far from obsolete: it is neither old, nor new, because Logo is Logo – a language – and it can be as current as we want depending on the use we give it, and we have found it invaluable in our Eiffel tower project.

This also reminds us of a comment made by Celia Hoyles in a visit that she and Richard Noss made to our school a few years ago; at the end of their visit, she said something like: “when I see your students working with Logo, my faith that Logo is still current, is reborn”. What we can add ourselves is that our experience with Logo is incredible. We believe that Logo is a fabulous tool: a tool for constructing, developing abilities, and learning how to think, that not many other software have matched. Many more novel and spectacular tools have appeared, but some are only used because they are fashionable or because others are using them – some users only want the latest and most externally appealing tools – without considering their true educational potential. We have tried a variety of software, but we only use that which we see as potentially fruitful (such as those we presented here: e.g. Google SketchUp and our blog), though we also believe in the value and importance of using a variety of approaches and modes of representation with which students can engage and interact (Wilensky, 1991).

We are not against evolution, but we are against an indiscriminate and unthought use of new software or ICT tools. And amongst the sophistication of much modern software, our students still like Logo best. Another one of this year's 13-year old students said the following:

*“Logo is a very cool and interesting software that teaches you to do different figures and you can create various procedures as if you were doing mathematics; you realize that with very little tools you can do lots of things and it really is all very intriguing all the things that you can do with these commands; it is fun and brilliant. And that is what I can say about this software that has taught me so much and the truth is that I would like to continue working with it.”*

Last year, with the following critical statement, one of our 13 year-old students explained why he preferred Logo to other more modern software:

“...because [in Logo] I can express myself... [whereas] I think buttons make human beings obsolete(!), that's why I don't like most modern software, because it is just about pressing buttons... What should be done is to let users create their own tools... But [modern software] is not good, it is making human beings learn not to think, whereas with Logo one has to think...”

Wow to whoever thinks that Logo is just a programming language – Logo is a philosophy; and just like mathematics is not just about numbers and is a way of thinking, Logo is also a way of thinking. Borrowing some of Papert's (1980, p.18) words, Logo “is a particular way of using computers, of forging new relationships between computers and people”.

## Acknowledgement

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