

# Intrinsic and extrinsic perspectives in 3d constructions

Maria Latsi, [mlatsi@ppp.uoa.gr](mailto:mlatsi@ppp.uoa.gr)

Educational Technology Lab, School of Philosophy, University of Athens

Chronis Kynigos, [kynigos@ppp.uoa.gr](mailto:kynigos@ppp.uoa.gr)

Educational Technology Lab, School of Philosophy, University of Athens

## Abstract

In this paper we report findings of a classroom research aiming at shedding light on 12 year-old students' construction processes as they worked with a 3d Logo / Turtle Geometry environment which we call 'MachineLab Turtleworlds' (MaLt). Illustrative examples of students' work are provided trying to examine in particular: a) the way the students used the software's functionalities of changing viewing angles throughout the construction processes, b) the interplay between the turtle metaphor and space visualisation through various viewing angles, c) the interplay between the perception of figures considered in relation to different viewpoints and in relation to their geometric properties. The analysis of the results brings in the foreground the dialectic relationship between the way the available viewing angle manipulation tools were used and the construction strategies followed, within the framework of particular tasks. It seems that the 3d space was experienced by the students through two distinct perspectives: an intrinsic and an extrinsic one. When the students were focusing on navigating and orientating the turtle, the 3d space was experienced through an intrinsic perspective, according to which the simulated space was viewed from inside, through the turtle's viewpoint. In this case the use of the body-syntonic metaphor was critical but yet not the same as conventional 2d turtle geometry. In contrast, when the focus was shifted from the management of the turtle's spatial movements to the construction of a graphic object, the 3d space was experienced through an extrinsic perspective, from the view point of an external observer who looked at the figural results of turtle's movement.

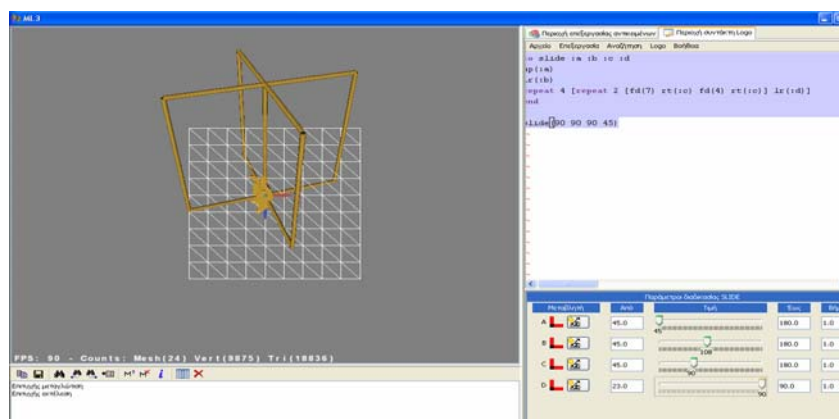


Figure 1: The 'MaLT' microworld

## Keywords

3d logo, constructionism, space visualisation, construction processes and viewing angle

## Theoretical background

The contribution of technology in the teaching and learning of geometry is perceived to be strongly linked with interactivity, multiple interlinked representations including symbolic ones, dynamic manipulations and dynamic visualisations (Laborde et al., 2006). However relatively little research has been carried out on the way the above distinct characteristics of digital media can be exploited so as to engage students in meaningful investigations especially as far as 3d geometry is concerned. Aiming at challenging students' intuitions and ideas concerning spatial visualisation and thinking (Presmeg, 2006, Arcavi, 2003) we developed a set of microworlds and a set of activities adopting a constructionist theoretical perspective (Kafai & Resnick, 1996). A distinct feature of the microworlds was that they were 'half-baked' (Kynigos, 2007), i.e. incomplete or buggy digital artefacts that they had to investigate how they work and to change and fix them.

Our pedagogical aim was to engage the students in navigating the turtle to construct graphical objects by means of Logo programming and dynamic manipulation of procedure variable values. Drawing with the turtle does not follow the conventions used so as to represent an object in a visually acceptable way e.g. depicting its external features in perspective. In contrast, it is based on analysing the visual characteristics and on explicitly addressing the geometric properties and structure of the object to be constructed along with the spatial relations of its elements. Students have to search for ways to reconceptualise 3d objects in terms that can be explained to the turtle through logo commands according to the distinct geometrical nature of turtle geometry. Turtle geometry is based on a different geometrical system to those usually associated with the learning of geometry and it has been characterised as differential by Papert (1980) and as intrinsic by Abelson & diSessa (1981). It's considered as differential as a given geometrical state of the turtle is fully defined by its relation to the turtle's immediately previous state. In a similar vein it is characterised as intrinsic in the sense that there is no need to refer to places outside the turtle's immediate vicinity when deciding on an input to a procedure to change turtle's state. Turtle geometry has been a field of long debate especially in relation to a) the degree children use the geometrical ideas embedded in it and b) the degree turtle geometry's intrinsic nature is related to other non-intrinsic geometries (Kynigos 1993). Researches seem to conclude that carefully designed Logo- based microworlds is an effective medium in offering rich mathematical experiences and encouraging inductive inferences from personal experiences which in turn can invite engagement in deductive thinking (Clements & Sarama, 1997, Kynigos, 1993). Moreover it seems that the turtle metaphor can be used to extend children's learning to include non-intrinsic geometry (Kynigos, 1992). However extending Turtle Geometry in 3d space offers a new perspective and raises new issues related to the way turtle metaphor is put to use and the way deeply rooted intuitions about experiencing space and locomotion can be exploited so as to make sense of geometric notions (Kynigos & Latsi, 2007).

MaLT is a 3d version of 'Turtleworlds', a turtle geometry environment with dynamic manipulation tools for variable procedure values (Kynigos et al, 1997). MaLT integrates various tools for manipulating dynamically both 3d objects (in the sense of Turtleworlds) and the viewing angle of the simulated 3d space. In particular, the viewing angle manipulation tools are designed to support visualisation processes, e.g help users discriminate between what is represented and how it looks, imagine all possible diagrams attached to a geometrical object etc. Although the functionalities available are designed so as to help users abstract 3d geometrical objects' properties and structure and to acquire a sense of space and shape constancy, this should not be taken for granted. In contrast, it should be a point of concern that needs investigation as it seems that three-dimensionality and animation present special challenges and there is little support that they are beneficial to graph comprehension (Tversky, 2005). The new visual representations available must be considered not only in respect to their appearance but also from the perspective of their use, in respect to the manipulations that can be carried out on them

as well as on the way other linked representations may be affected (Morgan et al., 2009). Viewing the images in the computer screen as 'signifiers' mediated by the conventional system in which they are created it is interesting to investigate how viewing an object from different perspectives would affect the actual process of getting to know the object and its spatial properties in the simulated 3d space. In particular building upon earlier researches (Kynigos et al., 2009) that took place within the framework of a European research and development project (ReMath, 2005-2009), the aim of our research was to investigate: a) the way students used software's functionalities of changing viewing angles throughout the construction processes, b) the interplay between the turtle metaphor and space visualisation through various viewing angles and c) the interplay between the perception of figures considered in relation to different viewpoints and in relation to their geometric properties

## The Computational Environment

MachineLab is a programmable environment for the creation and exploration of interactive virtual reality simulations developed within the ReMath project (ReMath, 2005-2009). MaLT was conceived as a constructionist microworld environment within MachineLab that extends the 'Turtleworlds' Turtle geometry to 3d geometrical space. Thus an extension of Logo commands in 3d space is provided including the two conventional types of turtle turns (Reggini, 1985): 'UPPITCH/DOWNPITCH n degrees' ('*up/dp n*') which pitches the turtle's nose up and down and 'LEFTROLL/RIGHTROLL n degrees' ('*lr/rr n*') which moves the turtle around its trunk/vertical axis. However the distinct feature of MaLT is that the logo-based Turtle Geometry is integrated with the dynamic manipulation of interactive graphical representations - a functionality characteristic of Dynamic Geometry Environments'. In particular, the dynamic manipulation tools available can be divided in two categories:

- a) dynamic manipulation of graphical figures by means of sequentially changing the variable values of the programs they create them through the use of specially designed variation tools (see the 1 dimension variation tool on the bottom right corner of picture 1).
- b) dynamic manipulation of the viewing angle of the 3d space: a) by using toolbar's buttons where the user can pick among 3 default views (front, side, top-down) b) by manipulating through mouse a specially designed vector tool, called the active vector, where the user can define either camera's direction or camera's position

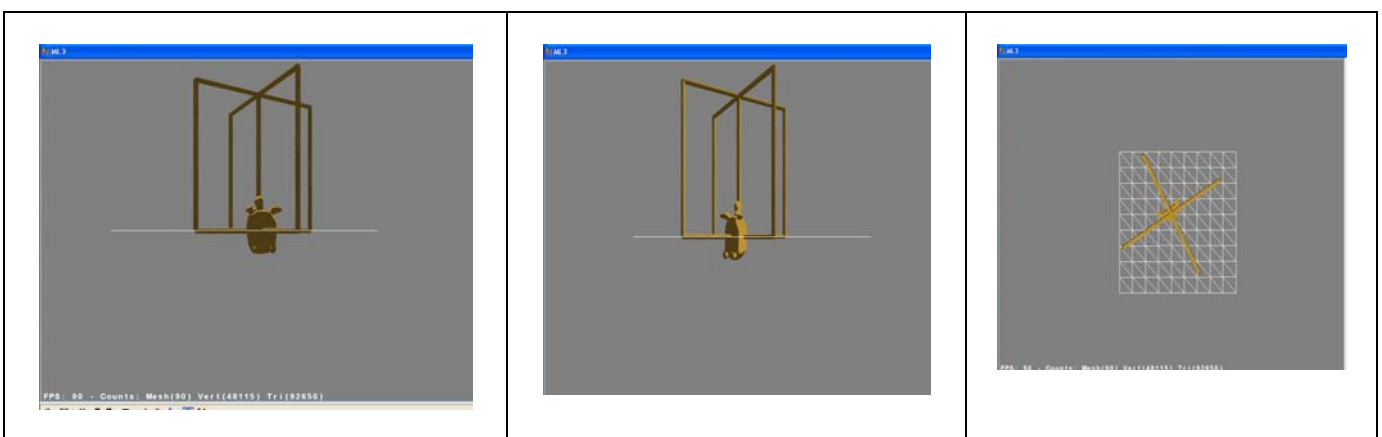


Figure 2: The 3 default views

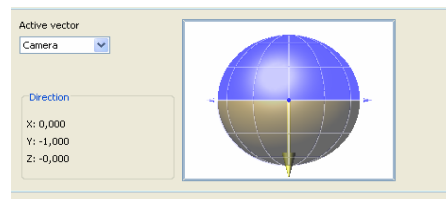


Figure 3: The active vector tool

## Methodology

The work reported in this paper is part of a design-based experiment in the sense that Cobb et al. (2003) have described it. The research took place in the 6<sup>th</sup> grade of a public primary school in Greece. The class had totally 16 teaching sessions with the experimenting teacher over two months. The tasks were designed to bring in the foreground issues concerning the mathematical nature of 3d geometrical objects and how they may be dynamically manipulated and transformed in mathematically meaningful ways. In particular we divided the activity sequence in two phases and we developed for each one of them a strand of two tasks. In task 1 the students were asked to navigate the turtle in such a way so as to simulate the take-off and the landing of an aircraft. In task 2 the students were asked to construct rectangles in at least two different planes of the graphical space of MaLt simulating adjacent walls of a virtual room. In the second strand of activities students experimented with half-baked microwords. In particular, in task 3 students were asked to use the 1dimention Variation Tool to control and experiment with the three variables of the half-baked microworld ‘movedoor’ that corresponded to different turtle turns so as to create the simulation of door opening and closing. The procedure was designed to have on purpose more than the variables needed. The students had firstly to decide which the role of each variable was and which values could be given to them. Then they had to build upon the half-baked microworld so as to develop a procedure that creates the simulation of a door opening and closing with the least possible variables.

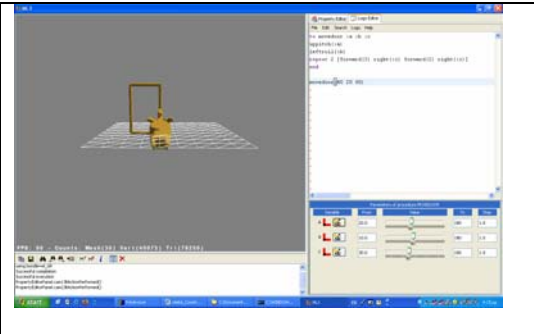
<pre> to movedoor :a :b :c   uppitch(:a)   leftroll(:b)   repeat 2 [forward(3) right(:c) forward(2)             right(:c)] end </pre>	
---	--

Figure 4: Simulating the opening and closing of a door and the respective logo code

In task 4 the students were asked to use the 1dimention Variation Tool to control the four variables corresponding to turtle turns in the ‘half-baked’ microworld ‘Revolving’ so as to create the simulation of a revolving door (see figure 5). The procedure was designed to have on purpose more than the variables needed. Students had firstly to decide which the role of each variable was and which values could be given to them. Then they had to build upon the half-baked microworld so as to develop a procedure that creates the simulation of a revolving door with the least possible variables. Finally the students were asked to extend the procedure of the revolving door in order to create a simulation of the fan of a watermill.

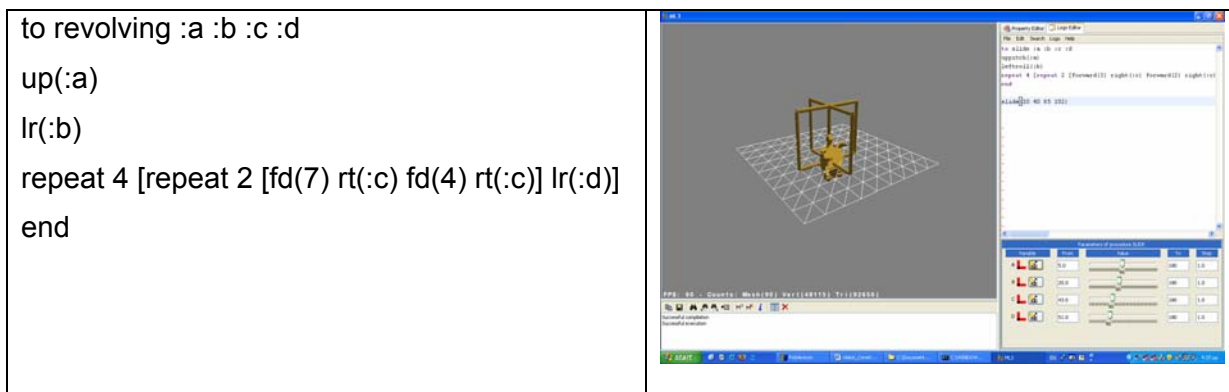


Figure 5: Simulating a revolving door and the respective logo code

In order to describe pupils' learning trajectories as they happened in real time we adopted a participant observation methodology while the main corpus of data included video-recorded observational data, researchers' observational notes as well as the sorting and archiving of the corpus of pupil's work on and off computer. Data were categorized in clusters of specific critical episodes that do not represent some quantifiable entity but are chosen to represent clearly the kind of activity that was going on in a specific time in the classroom. The analysis is still in progress while the results presented here are based on the work of one focus group focusing on the way viewing angle manipulation tools were used during the construction processes.

## Construction processes through different perspectives

The analysis of our results has shown that students' construction processes could be divided in two categories: construction processes through an intrinsic perspective and construction processes through an extrinsic perspective, depended on the focus and the way the simulated 3d space was experienced. This division reflects the two dominant perspectives people take on space (Tversky, 2005), an external one when they observe space and manipulate objects in it and an internal one when they explore an environment and when they navigate in it.

### Construction processes through an intrinsic perspective

Drawing with the turtle requires the formation of essentially novel methods of spatial orientation, where the reference point is not the position of the user's body but the turtle's body, relative to which the entire system of orientation may change. Thus during the construction process a critical point is the visualisation of space thought the different viewing angles of the turtle. Viewing space through the turtle's viewpoint is indirect and involves reflection about the turtle's viewing angle and orientation. However the degree of directness or not depends on the degree the body syntonetic metaphor is applied, in other words on the degree personal knowledge of movement in space is applied to the turtle's action. The more body-syntonetic turtle's motion is the more direct space visualisation and orientation become. Whereas in 2d Logo environments it is postulated that learning is aided by projecting one's own knowledge and experience of movement through space to the movement of the cursor on the screen (Fei, 1987), in the 3d logo environments body- syntoneticity is questioned (Kynigos & Latsi, 2007). However the results of the present research underline the importance of syntonising one's body with the 3d turtle – vehicle of motion in the 3d simulated space. During the construction processes of task 1 students preferred 'flying' the turtle along the z axis, that gave the impression of depth, and at a plane vertical to the display plane defined by the 2d computer screen. Moreover they kept on working on the default front view (although slightly slanted through the use of the active vector manipulation tool) even though they didn't have a clear representation of turtle's journey (see picture 4).



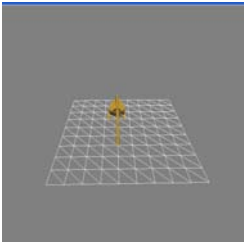
	Up(45) Fd(2) Dp(45) Fd(2) Dp(45) Fd(2) Up(45)
---	---

Figure 6: Simulating the taking-off and landing of an aircraft along the Z axis and the respective Logo code

It seems possible that children preferred flying the turtle along the z axis (that gave the impression of depth) while viewing the simulated 3d space from the default front view since this way they could more easily coordinate the various frames of reference (Wickens, 2005) present. In order to drive the turtle in a body syntonetic way students had to coordinate the following frames of reference: a) the ego frame, defined in terms of the orientation of the trunk or location of the observer b) the display frame defining in terms of the standard way of referring to things presented in the computer screen, where the right/left up/down directions are fixed, c) the world frame defined in terms of the fixed directions of 'up' and 'down', as a result of the gravitational effect and d) the vehicle frame of reference, defined in terms of the place and orientation of a moving entity, here the turtle. Flying the turtle along the z axis, the orientation of the vehicle of the motion, the turtle, coincided both with the orientation of students' body in the lived in 3d space and with the standard way of referring to the orientation of information on the computer screen. Students' comments corroborate this result. When asked why they preferred this kind of flight they replied: 'If we wanted to turn turtle right or left, we could see from our hands. If we wanted to turn it right, let's say, we would think where our hand is and we would send it to the right. '.

It is interesting that children are focusing more on body syntoneticity while not being sidetracked by the visual effects even though only an inclined line - corresponding to the 'taking off' of the turtle- was clearly visible on the computer screen. This result comes in contrast to the findings of other researches in the framework of 3d computational environments that have noted students' preference in working in a plane parallel to the computer's screen display plane (Kynigos & Latsi, 2006, 2007). Working in a plane parallel to the display plane is considered closer to students' experiences with 2d figures in school textbooks or with 2d Logo and would eliminate the convention used in the representation of the 3d space. However it seems that the kind of task and the metaphor used was of critical importance: the aim was not to construct just a slanted line or a geometrical figure but to simulate the taking off and the landing of the turtle - aircraft. In this framework the use of the commands *uppitch/downpitch* as well as the motion of the turtle along the Z axis that gave the impression of depth was rather more easily syntonised with everyday experiences and representations of flying aircrafts.

In the following tasks the students used extensively both the default viewing angle tools and the active vector during their construction processes. It could be suggested that the various viewing angle manipulation tools were especially used a) when a bricolage construction strategy was adopted (episode 1) b) when students were experimenting with specific aspects of the half-baked microworlds (episode 2). In the following episode students are trying to construct 'a wall' during task 2, giving commands to the turtle while using visual cues without having a clear strategy in mind. Their trial and error strategy is evident in the number of commands given to the turtle while trying to construct a parallelogram. It seems that every command is related only to the turtle's previous position and not to the whole construction process and the figure's geometric properties. When it wasn't visually clear if they had constructed a closed figure,

students did not resort to the geometrical object's properties (e.g. that the opposite sides of the rectangular figure should have equal lengths) but to the viewing angle manipulation tools so as to check if the figure was closed. Then they proceeded again forwarding the turtle little step by little step.

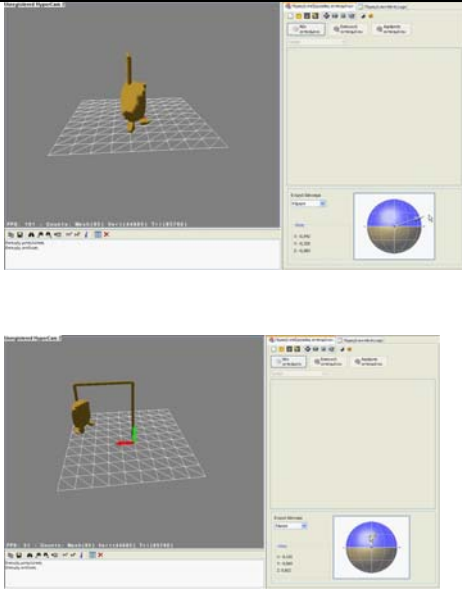
<p>Episode 1:</p> <p>S1: <i>Fd, more Fd, more...we should go it forward 0.5. Does it have 0.5?</i></p> <p>S2: <i>Wait, wait the turtle is here.</i></p> <p>S1: <i>Yes, but it hasn't touch the ground. Has it? Wait I have to see it.(he changes the view from side to front through the active vector tool and continuous forwarding the turtle)</i></p>		<p>Up(180)                  Dp(45)                  Dp(45)                  Fd(4)                  Rt(90)                  Rt(90)                  Rt(90)                  Rt(90)                  Rt(90)                  Fd(4)                  Bk(1)                  Bk(1)                  Bk(1)                  Fd(1)                  Fd(1)                  Fd(1)                  Dp(45)                  Rt(90)                  Fd(1)                  Fd(1)</p>
---	---	--

Figure 7: First column: Episode1, Second column: Changing viewing angles. Third column: The respective Logo code up to the point of the construction Episode 1 is referring to.

During task 4 students were initially experimenting with the values of the variables of the half-baked microworld 'revolving doors'. They had extra difficulties in finding out the role of the :d variable, which determined the measure of turtle's turning and respective position in the 3d space before drawing each successive door of the revolving door model. It follows that the d variable determined also the position of the four rectangle doors in the 3d space as well as their position in relation to one another. In the following episode students are conjecturing about the number of the visible rectangles (doors) if the value given to d is 720. However they do not find the front default view convenient and after testing all the available default views they choose to continue working with the top-down view active, where the number of doors created by the turtle was more clearly visible.

Episode 2:

S1: *Lets see how many doors there are if the value is 720 (he plays with the 1d variation tool changing the values of the d variable). Only one? This perspective is not convenient, I will change it (he activates successively all the 3 default views and opts for the top –down one)*

S2 *Yes, exactly like in the case of 360. It turns two rounds.*

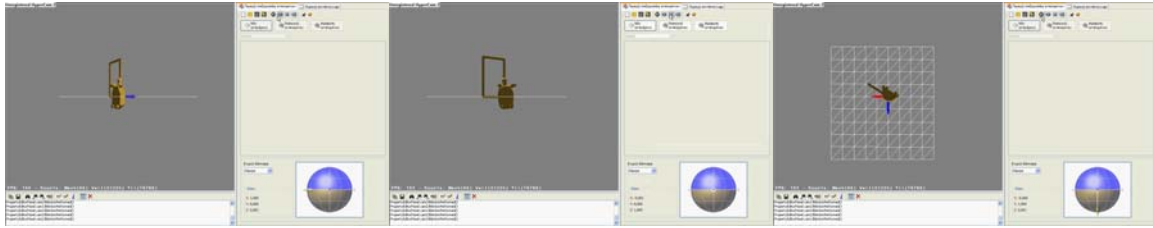


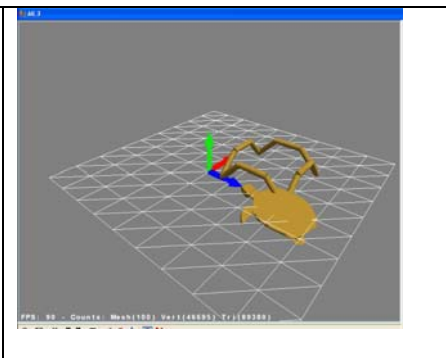
Figure 8: The 3 default views of the revolving door half-baked microworld when the value of the *d* variable is 720

In sum it could be argued that students have initially preferred a body sytonic way of navigating the turtle while opting for particular default views of turtle’s constructions according to challenges faced, e.g. a front view (although a bit adjusted with the active vector) during task 1, a side view (a bit adjusted again) during task 2, a top-down view during experimentation with variable *d* in task 4. It should be stressed that the preferred default views offered students 2d representation that possibly helped them focus on particular aspects of their construction. In this phase it was more important for students to explore the environment and navigate the turtle command by command taking advantage of the body-syntonic metaphor and viewing space through an intrinsic perspective. However as students’ construction strategies shifted to more analytic ones it seems that they ceased being so ‘immersed’ in the 3d space, a result that is treated in the next paragraph.

**Construction processes through an extrinsic perspective**

When using the turtle metaphor students have to pass from the management of turtle’s spatial movements to the construction of a graphic object (Fein & all, 1987), while making a distinction between the agent and the object, between the navigation of the turtle and the result of this navigation, the geometrical object. In parallel students have to coordinate two different view points: the view- point of the turtle which must be moved in an appropriate way so as to draw a figure and the view point of an external observer who looks at the figural results of turtle’s movement. The results of the present research suggest that as the activities unfolded, the students progressively adopted an extrinsic perspective of the 3d space, observing it as external viewers.

In the end of Task 2 there was some free time available and students spontaneously decided to try to construct a closed figure building upon their experimentation during Task1. Students were able to combine the flights they have previously constructed. Each taking-off and landing of the turtle was used as the building block of a ‘peculiar’ figure that came as result of four repeats of the initial turtle’s journey while turning turtle 90 degrees before each reexecution. It is also interesting -as it is evident in episode 3- that students adopted a more analytic strategy, visualising the whole turtle’s journey and explaining it to each other before entering commands to the microworld. Moreover when they returned to the microworld they did not execute the commands one by one but they inserted and executed a group of commands.

<p>Episode 3:</p> <p>S1: <i>Would you like to make a design?</i></p> <p>S2: <i>To make a circle?</i></p> <p>S1: <i>To go this way and then this way and again so</i></p> <p>S2: <i>Let’s make a triangle. First it goes this way and then it comes back. No, I have an idea, to insert 45 so as to go this way and then</i></p>		<p>Uppitch(45)</p> <p>Forward(2)</p> <p>Downpitch(45)</p> <p>Forward(2)</p> <p>Downpitch(45)</p> <p>Forward(2)</p> <p>Uppitch(45)</p>
---	--	---



<p><i>again 45 so as to go this way and then again 45 (they are showing on the screen and they are using their hands so as to simulate turtle's journey).</i>  <i>S1: Let's make a rhombus. So not right 45 but right 90 (So far they were talking to each other and now they return to the microworld inserting the commands)</i></p>		<p>Right(90)</p>
--	--	------------------

Figure 9: The closed figure and the respective logo commands that were executed four times so as to construct it

Another interesting point was that before starting their construction, the students adjusted their viewing angle through the active vector so that there was a clear sense of perspective of the simulated 3d space (see picture 7). They then continued working on their construction keeping this viewing angle stable. However this was not an occasional choice as the students followed the same strategy during the construction processes of the fan of the watermill during task 4: they adjusted their viewing angle so as to have again a sense of perspective (see picture 8) and they kept it stable throughout the whole construction process. When asked why they preferred this view the students just replied: *'It is more convenient because we can view the whole object.'*

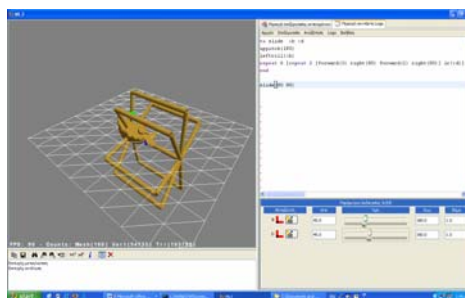


Figure 10: The view preferred during Task 4.

But the questions that arise are: Why students kept on working with a fixed view during 3d constructions that seem to necessitate a high degree of spatial visualisation and orientation. For instance wouldn't it be easier or more body-syntonic to change viewing angle in order to decide turtle's turning before each reexecution of turtle's flight during the construction of the closed figure (episode 3)? What were the reasons for this change as far as the use of the viewing angle manipulation tools is concerned as the activities unfolded? It seems that as students got progressively more accustomed to the 3d turtle's motion and the software's representational infrastructure they weren't so much concerned about body-syntonicity and that it was more important for them to have a clear sense of the threediness both of the simulated space and of the simulated objects. Constructing the simulation of a 3d object while viewing the simulated space in perspective was probably more realistic and familiar. However it could be also conjectured that students preferred a fixed view point during their constructions so as not to change position as observers and to have, thus, a stable point of reference which would probably be less cognitively demanding (Yakimaskaya, 1991). A fixed 3d view rather gave students a sense of space constancy, especially in cases that they adopted an analytic design strategy, as in episode 3, where they mentally visualised the whole turtle's journey before

executing the relative commands so as to construct the figure. Thus it could be argued that as the construction process became more complicated students preferred to view space from an extrinsic perspective, as external observers, focusing more on programming and geometric properties while taking into account the whole 3d space.

## Conclusions

The above analysis has tried to show that the way the available viewing angle manipulation tools were used was in a constant interplay both with task at hand and with the construction strategies followed. When the focus was on turtle's navigation and orientation in 3d space the body-syntonic metaphor came in the foreground while space was experienced through an intrinsic perspective: the user was immersed in space and was trying to view it from inside. In this case the students used various viewing angles which helped them face specific challenges and focus on particular aspects of their construction. The intrinsic perspective and the use of multiple viewing angles seems also to be adopted in case where a bricolage construction strategy was adopted, when the students had not a clear idea about the actions that should be taken and when the construction was progressing command by command through trial and error.

As the activities unfolded and as the students shifted focus from the management of turtle's spatial movements to the construction of a graphic object, they had started experiencing space through an extrinsic perspective, through the view point of an external observer who looked at the figural results of the turtle's movement. In this case a fixed 3d view was less cognitive demanding and offered students both a realistic effect of familiar objects, and space and shape constancy. Moreover a holistic/external view of the 3d space was in accordance with analytic construction strategies where the students were trying to visualise the turtle's journey taking into account the whole 3d space before executing commands on the computer screen. It was interesting that aspects of the distinction between the intrinsic and extrinsic characteristics of programming with the turtle metaphor in 3d space came in the foreground in a functional way through specific choices of viewing angles. It goes without saying that there were not clear cut borders between the two perspectives and that there were a lot of instances that students oscillated between them according to their construction focus. This research was a tentative effort in appreciating an aspect of the large spectrum of the representational potential of a specific 3d microworld in the context of constructionist activities. However a lot of further research is needed in order to investigate the way mathematical concepts can be integrated with spatial navigation and orientation in virtual environments, as well as in order to investigate the way highly visual microworlds, such as MaLt, can be used in educational design.

## References

- Abelson H. and DiSessa A. (1981). *Turtle Geometry: The Computer as a Medium for Exploring Mathematics*. Cambridge M.A.: MIT Press.
- Arcavi, A. (2003). The role of visual representations in the teaching and learning of mathematics. *Educational Studies in Mathematics*, 52(3), 215-241.
- Clements D. and Sarama, J. (1997). 'Children's mathematical reasoning with the turtle programming Metaphor', in English, L (ed.), *Mathematical Reasoning, Analogies, Metaphors and Images* Lawrence Erlbaum Publishers, Mahwah, pp. 313–338.
- Cobb, P., Confrey, J., DiSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher*, Vol. 32-1, 9-13.
- Fein, G. G., Scholnick, E.K. , Campbell, E.K. , Schwartz, S.S. , Frank, R. (1987). Computing Space: A Conceptual and Developmental Analysis of LOGO.. In R. E. Mayer (Ed.), *Teaching*

and learning computer programming: *Multiple research perspectives* (pp. 55-74). Hillsdale, NJ: Lawrence Erlbaum Associates.

Gutiérrez, A. (1996). *Visualization in 3-dimensional geometry: in search of a framework*. Paper presented at the 20th International Group for the Psychology of Mathematics Education, Valencia, Spain.

Harel, I. & Papert, S. (1991). *Constructionism: Research Reports and Essays*. Ablex. Publishing Corporation. Norwood, New Jersey.

Kafai, Y. and Resnick, M. (eds.) (1996). *Constructionism in practice: Designing, thinking and learning in a digital world*. Lawrence Erlbaum Publishers, Mahwah

Kynigos, C. (1993). Children's Inductive Thinking during Intrinsic and Euclidean Geometrical Activities in a Computer Programming Environment. *Educational Studies in Mathematics*, 24, 177-197

Kynigos, C. (1992). The Turtle Metaphor as a Tool for Children Doing Geometry. Learning Logo and Mathematics, editors: Hoyles C. and Noss R., Cambridge MA: M.I.T. press, 97-126.

Kynigos, C. (2004). A 'Black-and-White Box' approach to user empowerment with component computing, *Interactive Learning Environments*, 12 (1-2) 27-71.

Kynigos, C. & Latsi, M. (2006). Vectors in use in a 3d Juggling Game Simulation. *International Journal for Technology in Mathematics Education*, 13 (1),

Kynigos, C. & Latsi, M. (2007). Turtle's navigation and manipulation of geometrical figures constructed by variable processes in 3d simulated space. *Informatics in Education*, Vol. 6, No. 2, 359-372.

Morgan, C., Mariotti, M., A. and Mafei, L. (2009). Representation in Computational Environments: Epistemological and Social Distance. In *International Journal of Computers for Mathematical Learning*, 14 (3), 241-263

Newcombe, N. & Learmonth, A. (2005), Development of Spatial Competence. . In *The Cambridge Handbook of Visuospatial Thinking*, Shah, P. & Miyake A. (eds) (2005), Cambridge University Press, Cambridge, pp. 213 - 256

Papert, S. (1980). *MindStorms – Children, computers and powerful ideas*. The Harvester Press Limited, London

Presmeg, N. (2006). Research on visualization in learning and teaching mathematics. In A. Gutiérrez, & P. Boero (Eds.), *Handbook of research on the Psychology of Mathematics Education: Past, present and future*. Rotterdam: Sense Publishers.

'ReMath' - *Representing Mathematics with Digital Media* FP6, IST-4, STREP 026751(2005 – 2009)

Reggini, H. (1985) *Ideas y Formas*, Buenos Aires, Ediciones Galapago.

Tversky, B., (2005). Functional Significance of Visuospatial Representations. In P. Shah and A.Miyake (eds.) *The Cambridge Handbook of Visuospatial Thinking*. New York, Cambridge University Press: 1-34.

Wickens C., Vincow, M. & Yeh, M. (2005). Design Applications of Visuospatial Thinking: The Importance of Frame of Reference. In *The Cambridge Handbook of Visuospatial Thinking*, Shah, P. & Miyake A. (eds) (2005), Cambridge University Press, Cambridge, pp. 383-425

Yakimanskaya, I. S., Wilson, P. S. & Davis, E. J. (1991). *The Development of Spatial Thinking in Schoolchildren*, NCTM.