Representation systems of 3D building blocks in Logo-based microworld

Han Hyuk Cho, *hancho@snu.ac.kr*
Dept of Mathematics Education, Seoul National University, Seoul 151-748, South Korea

Hwa Kyung Kim, *indices@smu.ac.kr*
Dept of Mathematics Education, Sangmyung University, Seoul 110-530, South Korea

Min Ho Song*, mino@snu.ac.kr*
Dept of Mathematics Education, Seoul National University, Seoul 151-748, South Korea

Ji Yoon Lee, *lily1982@snu.ac.kr*
Dept of Mathematics Education, Seoul National University, Seoul 151-748, South Korea

Abstract

Logo has influenced many researchers and learners for the past decades as a 2D turtle geometry environment in the perspective of constructionism. Logo uses the metaphor of 'playing turtle' that is intrinsic, local and procedural. We, then, design an environment in which the metaphor of 'playing turtle' is applied to construct 3D objects, and we figure out ways to represent 3D objects in terms action symbols and 3D building blocks. For this purpose, design three kinds of representation systems, and asked students make various 3D artifacts using various representation systems. We briefly introduce the results of our investigation into students' cognitive burden when they use those representation systems, and discuss the future application measures and the design principles of Logo-based 3D microworld.

Keywords

Microworld; Logo; Representation system; Building blocks; Mathematics education;

1. Introduction

Papert introduced a virtual environment to create 2D figures using the two basic commands (rotate and forward), and learners create figures using the powerful metaphor of 'playing turtle.' Abelson and diSessa (1980) dubbed Logo as the turtle geometry, and compared it with the coordinate geometry, describing the features intrinsic, local and procedural.

Development of the spatial sense is one of the major goals in mathematics education. The Principles and Standards of the National Council of Teachers of Mathematics (2000) recommends the visualization and reasoning of 2D and 3D as a major capability to be enhanced by students. In particular, computer utilization is essential in visualization of the spatial sense in 3D.

The 2D Logo figures and 3D building blocks are introduced in Korean elementary mathematics textbooks. As for Logo, the Java Applet environment developed to enable making commands in Korean is introduced in mathematics textbooks for third and fourth graders. Students, then, learn about the metaphor of ‘playing turtle’, and are involved in activities to produce rectangles and equilateral triangles. Meanwhile, 3D building blocks are covered in textbooks for the second and sixth graders in Korea with a focus on daily activities. Here, 3D building blocks refer to three dimensional objects produced by connecting cubes of the same size. 3D building blocks are excellent manipulative to develop space sense with regard to mental rotations and visualization. Some mathematical activities applying 3D building blocks are finding the number of cubes piled up or looking for the arithmetic and geometric patterns.

If one thinks of the process of connecting cubes to construct 3D building blocks in a procedural manner, the metaphor of ‘playing turtle’ is useful for this process. In other words, if connecting one block to the next block is thought of as the ‘forward’ in the turtle’s gesture, choosing what planes of the existing cubes to newly connect to is the ‘rotate’ in the turtle’s gesture. In designing a Logo-based 3D microworld environment, the turtle creates a cube that surrounds its periphery whenever it moves, and the process of connecting cubes is represented by string action symbols. In this paper, we introduce the artifacts made by students as well as students’ construction activities using various 3D representation systems. We examine the cognitive burden students bear when they use 3D building blocks representation system, and examine situations where each representation system is effectively applied.

2. Theoretical perspective

Ackermann (2004) argued that in comparison of Papert’s constructionism and Piaget’s constructivism, although both theories are of the same view, the roles of the media are more emphasized in constructionism. Here, the media being emphasized is a physical construction environment for mental construction. Likewise, Kafai and Resnick (1996) said that the core of constructionism is the mental construction through physical construction.

- Constructionism is both a theory of learning and a strategy for education. It builds on the “constructivist” theories of Jean Piaget, asserting that knowledge is not simply transmitted from teacher to student, but actively constructed by mind of the learner. Children don’t get ideas; they make ideas. Moreover, constructionism suggests that learners are particularly likely to make new ideas when they are actively engaged in making some type of external artifact-be it a robot, a poem, a sand castle, or a computer program-which they can reflect upon and share with others. Thus, constructionism involves two inter-wined types of construction: the construction of knowledge in the context of building personally meaningful artifacts (Kafai and Resnick, 1996).

The computer environment to implement constructionism must be the one where learners can construct the artifacts they want and construct knowledge through the very construction activity. In other words, through this learning by making, learners naturally come to know of powerful ideas through the activity of physical construction.

Bruner (1966) remarked that children's intelligence development is that of enactive, iconic and symbolic representations and that of adjustment capabilities amongst them. In Logo, learners produce the turtle's movement on the screen through the representation of symbolic commands that coincided with the turtle's gesture and their own gesture and identify the outcomes in an iconic manner. In other words, visualization in Logo is the turtle's enactive representation and the iconic representation of the outcomes of the vestige. Such a metaphor of 'playing turtle' is more dynamic than that of a static figure, which connects the external media and the individual's mind in a friendlier manner (Christou et al., 2007b).

Arabic numerals and Roman numerals are different representations for the same numerals, but are starkly different in the cognitive and calculation processes. The difference can be seen in Roman numerals versus Arabic numerals, where the same number is represented by different numerals. Cognitive operations are not independent of the symbols that instigate them (Gonzalez and Kolers, 1982). Similarly, Norman (1993) said that the cognitive burden differs depending on the representation system. Resnick and Silverman (2005) still emphasize the roles of programming in the computer fluency aspect. But the programming environment must be improved for the convenience of users. That is why we suggest heterogeneous representation systems to produce procedural building blocks using string symbols in the Logo-based programming microworld.

The plane geometry can be approached using different representations. Abelson and diSessa (1980) introduced vector geometry, a different system from turtle geometry. It is a method of producing a curve which is similar to a function graph in the vector perspective into the Leibniz style. In this case, the turtle moves to the extent of the given vector to the direction of X-axis and Y-axis. This method, thought local and procedural like the metaphor of 'playing turtle' is not intrinsic as the direction of axes are already set. These two different representation systems produce the gesture of 'forward' and 'rotate' using the metaphor of 'playing turtle', and construct local and procedural plane figures by connecting these gestures. Students might have different cognitive burden in these two different representations, which have their own strengths.

3. Design principles of 3D building blocks.

3.1 JavaMAL microworld

The JavaMAL (http://www.javamath.com/class) microworld we are to use is a web-based environment, where Logo and DGS (Dynamic Geometry System) are integrated, and the text command can be stored, modified, and communicated on the attached web-board and executed there to reconstruct mathematical objects. Programming in JavaMAL microworld is mostly text-based, but commands can also be made using a mouse for convenience. The Input in both English and Korean is enabled in programming, and the Logo commands (forward and rotate, etc.) enable not only the turtle but also other objects (point, turtle tile and turtle net) to move.

In JavaMAL microworld, forward and rotate commands are the basic movements in the turtle geometric perspective and the move commands in the horizontal and vertical directions are the basic movements in the vector and coordinate geometric perspective. Those movements can be mathematically represented using the followings.

$$\Delta s,\ \Delta\theta\ /\ \Delta x,\ \Delta y$$

Abelson and diSessa (1980) said that turtle geometry and vector geometry are two different representations of the same thing, and the two representations of the same thing can often lead to insights that are not inherent in either of the representations alone. Expecting for this insight, we are to design building blocks where the two different representations (ver. A, ver. B) are both enabled.

3.2 3D Building blocks in JavaMAL

Let us call the command to make the square face with the metaphor of 'playing turtle', string 'm,' and the command to fold the dihedral angle between faces, another string symbol, '<, >.' One can make a cube by folding the square faces (shown in Figure 1), and let us represent folded cube as another string symbol, 's'. Then, 'ss' becomes a command to continuously connect two cubes. Next, newly assume the commands to changes the direction of the cube to be connected as the turtle commands of L(left), R(right), U(upward) and D(downward). Then, the representation of 'ssRssLss' will have the cubes connected in the order as in Figure 1 to represent building blocks. True, the same strings can enable squares to be connected instead of cubes.

Various building blocks can be produced using this representation system. For instance, Figure 2 is an artifact called 'swimming baby'. We produced an arm, a leg and a head by connecting several square faces and cubes. Cubes were made by connecting square face (m), and square face (m) and cube (s) can be simultaneously connected as in Figure 3. A railway and a train can be made using square face (m) and cubes(s), respectively. And pieces of the SOMA cube can be made into cubes (s), each piece can be connected to invisible square face (m), and the connected invisible faces can be folded. Note that this would generate the animation effects to connect the SOMA cubes (shown in Figure 3).

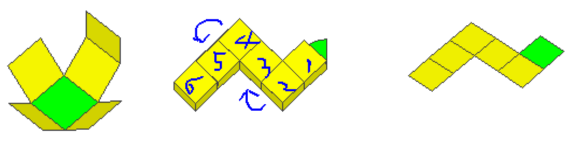


Figure 1.

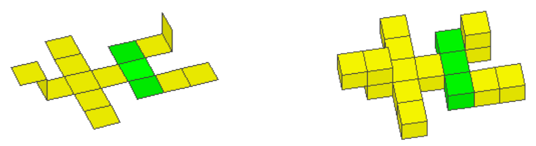


Figure 2. Swimming baby

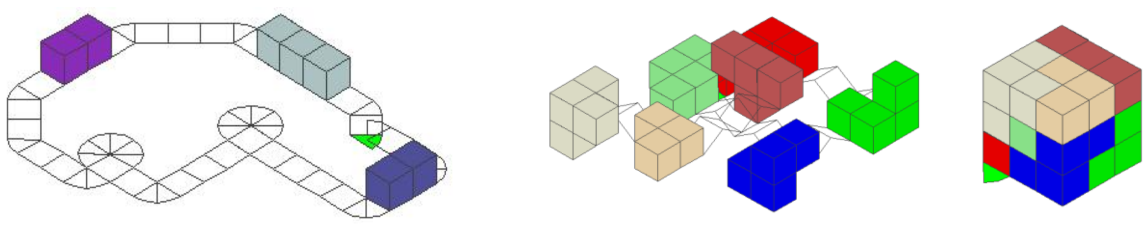


Figure 3. Train and SOMA cube

3.3. Representation system related to the metaphor of 'turtle playing'

Some researches try to represent 3D object using its frames, where each frame of the 3D object is drawn by placing the turtle in 3D space. For instance, MaLT (Kynigos and Latsi, 2007) and VRMath (Yeh and Nason, 2004) produce the turtle's gesture in 3D and visually show the vestige of the turtle (shown in Figure 4, Figure5, respectively). These environments were proposed by

Abelson and diSessa (1980) as turtle geometry view point in which cubes are piled up following the commands of the turtle. Let this kind of representation system be ver. A.

In ver. A, only ‘s’ makes cubes with the turtle moving forward, and commands ‘R, L, U, and D’ change the head direction of the turtle without making cubes. Ver. A uses the metaphor of ‘playing turtle’. However, although intuitively understandable and successful outcomes are generated by the metaphor of ‘playing turtle’ in 2D, difficulties might arise in connecting them to our daily activities in 3D. A turtle’s act of turning right and turning left on a 2 dimensional horizontal flat plane befits our daily context, and turning upward and turning downward are influenced by gravity as they are equivalent to walking on the vertical wall, which has a different context from our daily one. Likewise, Morgan and Alshwaikh (2008) said that students find it difficult to connect the daily gestures to the metaphor of ‘playing turtle’.

- While students were making use of similar gestures, we found that the meanings they tried to make with the gestures were different from those anticipated, and that the “playing turtle” metaphor did not easily transfer into the 3 dimensional context (Morgan and Alshwaikh, 2008).

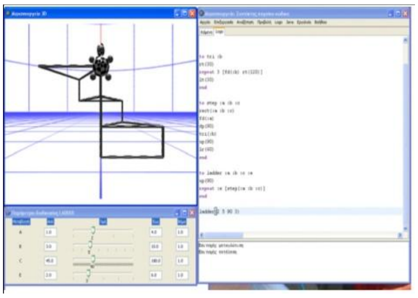


Figure 4. MaLT

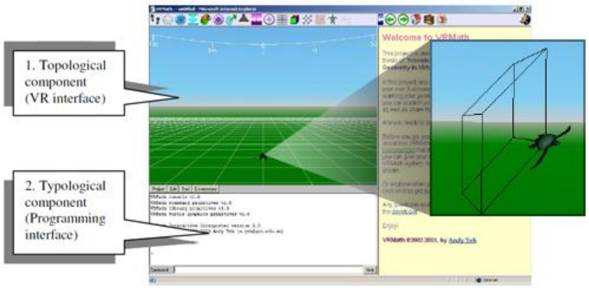


Figure 5. VRMath

3.4. Representation system related to the metaphor of ‘mouse attachment’

There are several computer environments that provide the proper environments for making 3D building blocks. For instance, DALEST project (Christou, et al., 2007a) or Freudenthal Institute’s WisWeb (http://www.fi.uu.nl/wisweb/en) is a computer environment to produce and visualize 3D building blocks (shown in Figure 6, Figure 7, respectively). When you click on any face of the cubes in Figure 6, you can attach another cube to that face. Here, we think of a command system using the metaphor of adding blocks by mouse clicking. Let’s choose one of the six faces of *(n-1)*-th cube and connect the *n*-th cube to be added to that chosen face. This is similar to the aforementioned vector geometry: three axes of a space are preset and strings that connect blocks along the axis are preset. For instance, blocks are added in such directions where string

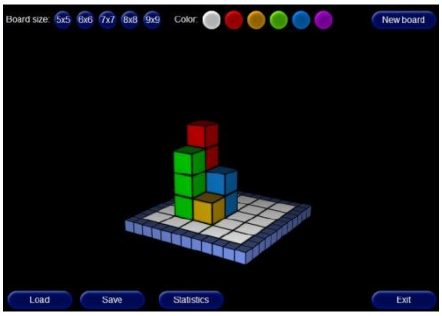


Figure 6. DALEST Project, Cubix Editor

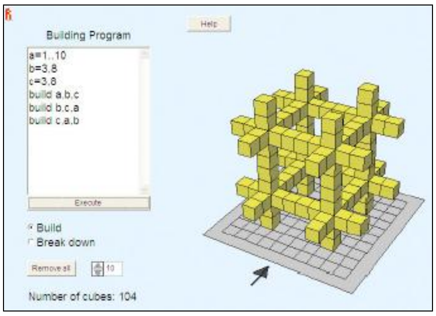


Figure 7. WisWeb

‘s, b’ is in the X-axis direction, ‘r, l’ is in the Y-axis direction, and ‘u, d’ is in the z-axis direction. We call this kind of representation system ver. B.

3.5 Three kinds of representation systems of 3D building blocks.

Let us be reminded that the 3D procedure is hard to be connected to our daily ‘gestures’ in ver. A. We would better think of representation mechanism more similar to our daily gestures. Of course, the representation system, this time, must be procedural. Kynigos and Latsi (2007) mentioned that when we move in real 3D space the up and down directions are usually stable because of gravity. Moreover, we walk in a 2D horizontal plane while the 3D turtle moves in different planes in 3D space. In a similar context, the reason why it is difficult to connect daily gestures to building blocks is because of ‘turn upward(U), turn downward(D)’. Thus it seems nice to introduce the metaphor of ‘elevator’, or ‘layers’ (combined version of ver. A and ver. B), by considering several horizontal and vertical layers. Within the horizontal layer, turtle moves and rotates related to the action symbols ‘s, L, R’. But within the vertical layer, turtle move upward or downward related to the action symbols ‘u, d’, which represents the construction of a cube one level upward and one level downward. In other words, we use ver. A for a turtle movement in horizontal direction, and ver. B for a movement in vertical direction. We might compare it to a scene where a turtle is on an elevator to mover around within a building to create building blocks. We call this representation system ver. C.

Ver. C is a form that combines ver. A’s commands ‘s, L, R,’ and ver. B’s commands ‘u, d.’ And ‘u, d’ in ver. C are equal to u = ‘UsD’, d = ‘DsU’ in ver. A. The three kinds of representation system can be summarized as in Table 1. We guess that the representation system ver. C might relive the difficulties among students having difficulty in ver. A.

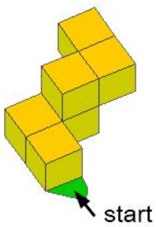
Movement	ver. A	ver. B	ver. C
Horizontal	Move Forward (s) Turn Right (R) Turn Left (L)	Move Forward (s) Move Backward (b) Move to the Right(r) Move to the Left (l)	Move Forward (s) Turn Right (R) Turn Left(L)
Vertical	Turn Upward (U) Turn Downward (D)	Move Upward (u) Move Downward (d)	Move Upward (u) Move Downward (d)
	ssRsUsDsLs	ssrurs	ssRsusLs

Table 1. Three kinds of representation system of 3D building blocks

4. Design activities with 3D building blocks.

Papert (1980) said that conversation with a computer must be as natural as learning French by living in France instead of learning a foreign language in a classroom. Resnick (2008) said that among Papert’s ideas, he was significantly influenced by ‘hard fun' and ’lower floor and higher ceiling.' We derive from the same idea. We expect that through the designing activity of making their own artifacts in the representation system of building blocks, the students could feel ‘hard fun,’ reflect on their own building block procedure, and build up their knowledge.

4.1 Free design activities of 3D building blocks

We introduced the representation system of 3D building blocks and carried out a design activity to enable them to make their own artifacts in the training program for gifted middle school students and in the teacher training program at Seoul National University, both of which were conducted in the summer of 2009. We introduced three representation systems to the participants, who then created the artifacts whatever they wanted using 3D building blocks, and gave each artifact a name. Figure 8 is an artifact named "P-51B Mustang Flying Fighter" made by 7th grader, and Figure 9 named "A Sailing Ship" and "Yu-na Kim, the Figure Skater" respectively made by mathematics teachers.

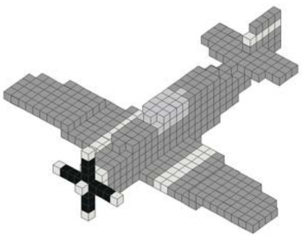


Figure 8.

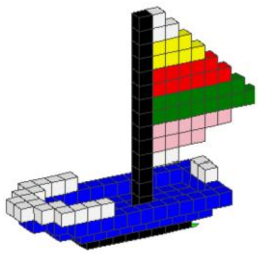


Figure 9.

4.2. Study

Students tend to prefer ver. B and ver. C to ver. A when they performed free design activities, but they prefer to use ver. A under the circumstances where they had to use recursive algorithms or production rules. As such, we conducted a simple test to see which one between ver. A and ver. C was regarded as the more difficult representation system by participants and how difficult it was in which circumstances. Ver. B was excluded in the experiment as it was believed to be meaningless in terms of measurement as the turtle's head direction is consistent.

We conducted a simple test for a total of 26 students in the first and the second grade in middle schools who were taking the training program for gifted students at Seoul National University in the winter of 2009. We only analyzed the data of 24 students leaving out two students because their correct answer rate is below 20% and the average time spent for each question was less than five seconds, which showed they had not taken the test seriously enough.

The test was conducted on a computer basis in a classroom where PCs were already set up. Questions were posted on the Web, and students' answers and the time spent for each question were recorded using the LRS (Learner Response System) installed in a JavaMATH server. Participants could start the test by pressing the start button, spend up to 60 seconds per question, and move onto the next question once they chose an answer.

There are fifteen ver. A problems and fifteen ver. C problems consisting of heterogeneous string representations. Thus students answered a total of 30 questions, and the questions were presented to students randomly without order. They scored one point if they got it right and zero point if they got it wrong, and each version had the full score of 15 points.

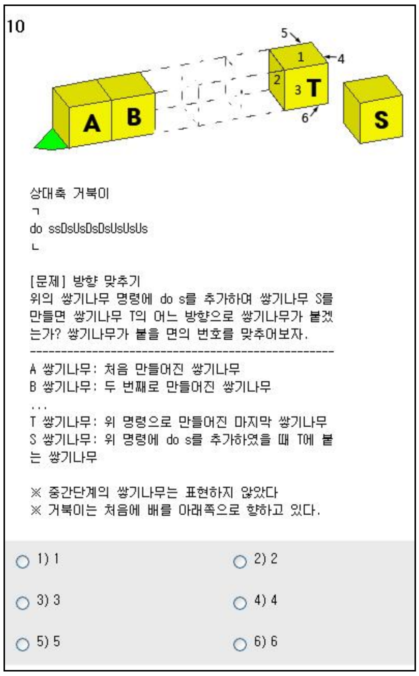


Figure 10. Example of the problem

As shown in Figure 10, the string representation and the resultant 3D building blocks were omitted in the middle part and only the first and the last cubes were suggested in each test problem. We then asked the participants where it would be attached on the last cube when a new cube was added. Participants were asked to input their answer by choosing one of the six faces of a cube. For instance, we suggested 'ssDs' to students and then asked them where the next 's' would be connected.

We analyzed the total scores and the total time spent by the 24 students in each ver. A and ver. C test using the paired sample t-test. As a result, they scored lower points in ver. A than in ver. C, but it took longer responding time in ver. A. The outcomes are summarized in Table 2. The total scores and the total time spent showed significant differences between the two 3D representation systems when the statistical significant level is 0.05. (t_1=-0.15, p_1=.000*; t_2=4.026, p_2=.001*)[1]. Out of the 15 questions, it was also reaffirmed that a question related to the combination of vertical and horizontal gestures showed significant differences. Moreover, the mixture of the vertical (D, U) and horizontal (R, L) action symbols raised the degree of difficulty in ver. A.

	Ver. A (turtle metaphor)			ver. C (elevator metaphor)		
	Mean	SD	%	Mean	SD	%
Assignment done (max.15)	10.46	3.16	69.73	13.54	2.32	90.27
Total time (s)	397.17	128.75		333.21	127.32	
Time per assignment (s)	37.97			24.61		

Table 2. Result

4.3 Discussion

As guessed earlier, the turtle's gesture in 3D was confusing to participants when the commands of 'turn upward and turn downward' were mixed. As emphasized by Papert (1980), the representation of building blocks using the turtle metaphor has educational significances in that various mathematical circumstances can be thought of by serving as an 'object to think with'. However, as it is not naturally connected to daily gestures, it might generate cognitive burden. As such, a representation system easily relatable to daily gestures could enable students to maintain a more comfortable status. In spite of this, we need to acknowledge that things that are easily accepted cognitively may not be always the best as 'hard fun' and 'higher ceiling' were emphasized by Papert. We need to use each version properly as considering which version is the most useful in which circumstance.

Students usually used ver. B and ver. C when they were asked to create whatever they want, but we observe some situations where they found ver. A more useful. Using the 3D representation system of building blocks, we were able to think about the "metaphor of growing" through the string substitution rule s='ss'. We observed the students who conducted s='ss' string manipulation after they made their artifacts freely. For example, we observed a student who made a soccer player with ver. A trying to increase its size twice by extending in the vertical and horizontal direction twice. Note that ver. A command has only string 's' which generates 3D building blocks (shown in Figure 11). However, those who made a soccer player with ver. B or ver. C commands could not use the string substitution rule s='ss'. Through such string manipulation, the students were actually able to feel the power of turtle metaphor system ver. A.

[1] t_1 and p_1 represent t-value and p-value respectively in total score comparison of ver. A and ver. C, and t_2 and p_2 represent t-value and p-value respectively in total responding time comparison of ver. A and ver. C. Asterisk means p-value is less than 0.05, which is a statistical significant level.

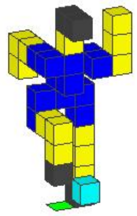
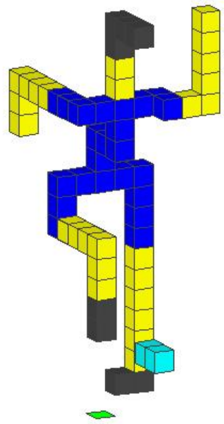
do_0	do_1
	

Figure 11. Soccer player when string ’s’ is manipulated to ‘ss’ in ver. A

5. Closing remarks

5.1 Mathematics Education

The number of blocks related to the substitution rule s=‘ss’ has the pattern of geometric sequence. Then, what can one do to make the arithmetic sequence pattern and the Fibonacci pattern? Moreover, we can create a self-repetitive image or a recursively growing form by the string substitution, and we can think of the string manipulation with regards to probability. For example, ‘z’ in ver. A is a dummy variable with no special meaning. Once we make a flower by building blocks with a dummy variable, we can change the length of its leaves using the probability rule. After including z command to the both-side leaves, command z=‘ztscu, z[2]’, and differentiate it once. Then, the probability that the length of leaves gets longer (ztscu) and the probability that the length of leaves stays the same (z) become 1/2, respectively. That is, upon each execution, the form of building blocks is to change (shown in Figure 12).

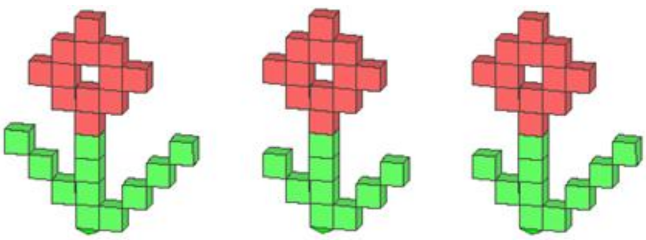


Figure 12. String manipulation of flower leaves with regards to probability

Furthermore, we can think of a case where the probabilities of z=‘sz’ and z=‘LszR’ are p and 1-p, respectively, and think of a case where the differentiation is made in each level by *n*th times. Here, the probability for the last cube to be positioned in a certain spot is related to the binomial distribution. This would enable mathematical knowledge to be connected to a physical

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[2] The string symbol ‘t’ means to make transparent building blocks and the string symbol ‘c’ means to make colorful building blocks.

substance. Trinomial distribution can be made using the same method. We expect that such an activity would facilitate learners to more easily take mathematical knowledge.

Next, learners would be able to intuitively understand mathematical features of 3D figures like Euler's polyhedral formula through building block activities. In fact, gifted middle school students intuitively explained why the value of 'V-E+F' is maintained through the activity of procedural connection of building blocks. As seen step1 from Figure 13, as one cube is added, two faces disappear as they overlap, and 4 edges and 4 vertexes disappear as they overlap. Therefore, if Euler formula works in each cubes (V_1-E_1+F_1=2, V_2-E_2+F_2=2), Euler formula also works in Euler formula (V-E+F=2) in the connected circumstance because overlapped Euler characteristic (V(4)-E(4)+F(2)=2) disappears although the sum of each building blocks' Euler characteristics is 4, which is the sum of V_1-E_1+F_1=2 and V_2-E_2+F_2=2. However, when there is an overlap of the existing parts as in step 2 and step 3, one can intuitively understand that the formula does not work. In fact, middle school students discovered the counterexample of the torus form themselves. In the torus form, 4 faces are to disappear, so they intuitively explained that V-E+F=0.

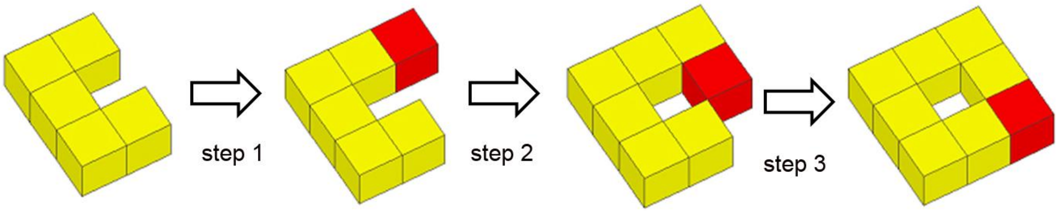

Figure 13. Euler's formula and an counterexample (exception)

5.2 Concluding

As in Logo, we consider the representation system of 3D building blocks as an intrinsic, local and procedural system. In addition, we propose 3 different types of representation systems (ver. A, B, C), and introduced several artifacts made by learners using these representation systems. We also figure out the difference of difficulties felt by students in different versions, and identified the difference through the structure of the representation systems. More researches are needed to design a desirable representation system of 3D objects.

Also more researches are needed on the three kinds of proposed representation systems. First of all, there is a need to systematically research cognitive burden of learners on different versions. It is also necessary to devise more systematic investigation techniques to measure the level of difficulties felt by learners, and to investigate how difficult learners would feel about certain occasions. Moreover, based on the researches about the level of difficulties felt by students, one needs to find out when each version is useful. Researches that utilize 3D building blocks as mathematical education tools in a classroom and investigate into mathematical effects are required as well. We believe that intrinsic, local and procedural representations of 3D building blocks and physical configuration activities under this representation system are needed for an educational goal of developing the sense of space in mathematical education.

6. Acknowldegement

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