On the design of Logo-based educational microworld environment

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Abstract

We study to design educational Logo-based microworld environment equipped with 3D construction capability, 3D manipulation, and web-based communication. Extending the turtle metaphor of 2D Logo, we design simple and intuitive symbolic representation system that can create several turtle objects and operations. We also present various mathematization activities applying the turtle objects and suggest the way to make good use of them in mathematics education. In our microworld environment, the symbolic representations constructing the turtle objects can be used for web-based collaborative learning, communication, and assessments.

Keywords

Microworld; Logo-based environment; Symbolic representation system; Mathematics education;

1. Introduction

Papert (1980), who had proposed 2D Logo, observed that students enjoy drawing pictures. He viewed the core elements of drawing as lines and angles. Accordingly, he proposed a Logo microworld where a virtual turtle can draw pictures with lines and angles using the given embodied commands. That is to say, the Logo is designed as a virtual environment that can become a mental playing ground similar to a learner's activity of direct bodily movement. Calling this Logo-based activities as turtle geometry, Abelson and diSessa (1980) compare it to coordinate geometry taught in existing math courses at school. Logo is also a representation system that students use to express, as if speaking, various mathematical objects, physical objects, or situations by using commands that are familiar to them. Logo goes further to become a powerful tool for expressing, investigating, and analyzing mathematical objects or situations. Centered on action symbols of forward and rotate, Logo is maintains its consistency while changing representation systems according to various demands, the Logo proposed by Papert is an excellent representation system for drawings on 2D-planes. Also the 2D L-system introduced by Lindenmayer transforms forward and rotate command into f, + symbols respectively while grafting into Logo environment, becoming an experimental tool for natural phenomena, fractal investigation, and others through the resulting differentiation of representation (Wagon, 1991). Studies have also been made showing several turtles at once in order to reenact physical phenomena or traffic situations (Klopfer, Colella and Resnick, 2002). Moreover, as spatial sense of 3D-figures came to be regarded as important, a turtle moving in 3D has also appeared (Morgan and Alshwaiks, 2008).

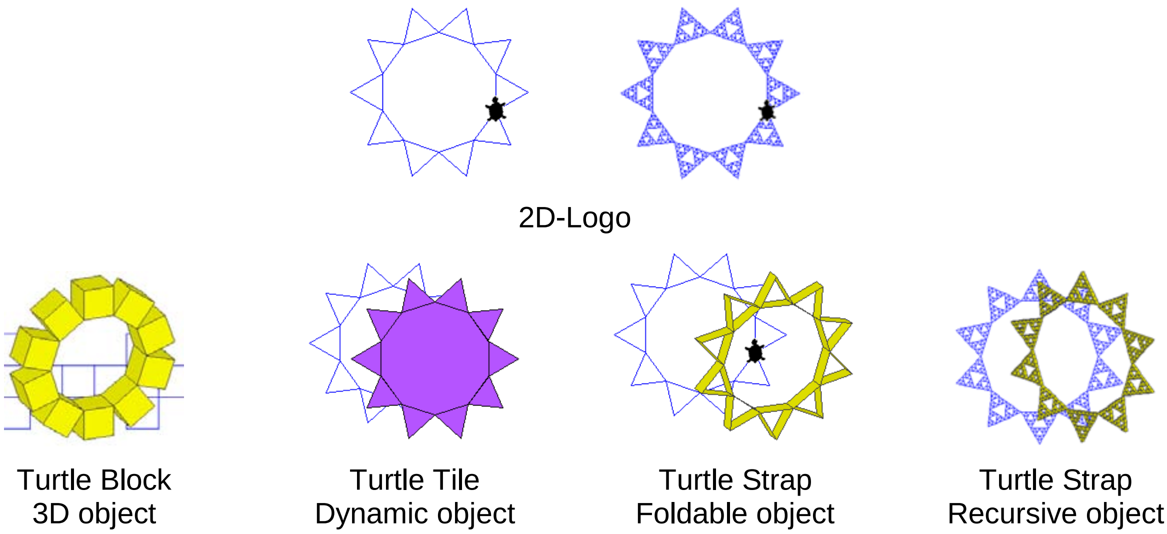


Figure 1. Turtle objects and turtle operations

The basic philosophy of Logo is to express the core of an object in the most basic language. Therefore, the application of Logo to various situations requires that one finds the core that represents the situation and that the language representing the core must be natural as well as meaningful to the learner. Extending the turtle metaphor of 2D Logo, we design simple and intuitive symbolic representation system that can create several turtle objects and operations as shown in Figure 1. The turtle objects and operations are powerful tools to construct mathematical situations for mathematization, and we suggest the way to make good use of them in mathematics education in this paper. Taking note that representation systems are based on symbols, and we also examine how the symbolic representation system constructing the turtle objects and operations can be used for web-based collaborative learning, communication, and assessments.

2. Design principle of turtle objects

Mathematical objects or situations expressed using symbolic representation system become important tools for mathematization. The process of expressing, analyzing, and interpreting objects using symbols is a must needed process in the abstraction and generalization of mathematics. Mathematical communication through symbols also enables the internalization of mathematical knowledge, allowing learners to go beyond a mere acquisition of knowledge. However, it is also true that the difficulty of such symbolic representation also becomes an obstacle for learners. Stating that knowledge with stories appropriate to situations are more valuable to learners, de Jong and Monica (1996) emphasize the importance of situational knowledge. Thus, they minimize the obstacles to learners by introducing commands that are meaningful to them as symbols while also facilitating various activities by examining symbols from a powerful-idea perspective. The translation of Logo's basic commands into meaningful symbols enables an approach to Logo through the perspective of operating symbols, and such an approach may become the link to mathematization of elementary and secondary mathematics.

Van der Meij and de Jong (2006) study representational activities through various methods of media. Mentioning how a single representation aids the understanding of another representation, they state the need for understanding previously abstract concepts more deeply through translation of representations. They also mention that the addition of meaningless information to users may only cause greater confusion. In other words, what is demanded is the activity of representing mathematical objects through symbols that are meaningful to the learners. Wilensky (1991) mentioned whether something is abstract or concrete is not an inherent property of the thing, but rather a property of a person's relationship to an object. And good concrete manipulatives are those that aid students in building, strengthening, and connecting various representations of mathematical ideas (Clements, 1999). This requires an environment in which one can find the core of one's representational object and method while representing it in a language that is meaningful to learners. As the main example of such an environment, Logo is a microworld that considers segments and angles between segments as the core elements of drawing on a 2D-plane and makes a turtle draw them. We now propose a representation system that can become a learner's language like Logo commands on an extension of the Logo turtle's action symbols.

Operating symbols can be applied to various situations that may be represented with a limited number of commands. Figure 2 is a game in which an avatar is moved in four directions to move the balls to a designated place. This game is made simply of keyboard operations, but this game can also be made of symbol manipulation as well. The symbols represented in the center represent the alien's movement in four symbols of u(p), d(own), r(ight), l(eft).

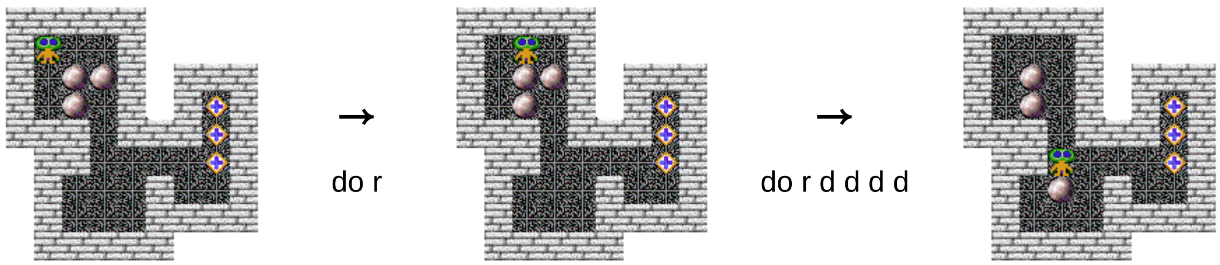
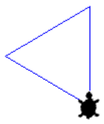


Figure 2. Symbolic representation in a game

The motion of Rubik's cube can be represented by 6 symbols: R, L, F, B, U, D. When we discuss the way of manipulation of Rubik's cube, we are apt to use these symbols. Eidswick (1986) approached to solution of Rubik's cube by using those symbols algebraically. Thus even when they are games, it is possible to represent complicated situations using simple symbols to which users invest meaning; and the case also holds true in reverse. That is to say, symbol-based microworld always has to have the use of operating symbols in mind; it must achieve an environment in which users can become familiar to operating symbols and engage in activities of representing complicated objects in simple basic pieces. It is possible to execute and represent symbols using ‘do’ command in JavaMAL microworld. For example, we can replace the command ' fd ' to ' f ' and ' rt ' to ' < ', respectively, as you can see in Figure 3(forward default value is 10, and rotate default value is 120).



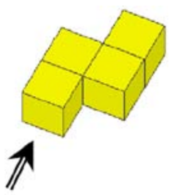
fd 10; rt 120; fd 10; rt 120;fd 10; rt 120;
do f < f < f < ;

Figure 3. Triangle (2D-figure) and two different representations

2.1. Turtle block

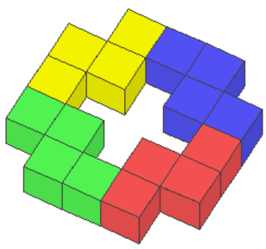
The Logo proposed by Papert (1980) is an excellent representation system of drawings on a 2D-plane. Due to the development of media, there is increased activity of operating 3D-figures as well as greater emphasis on its importance. It seems nice Logo’s representation system evolve into a system that can represent 3D-figures naturally in turtle language. The use of complicated commands will clearly enable the creation of more realistic 3D-figures, but it would be difficult to expect their use as methods of communication. When learning Logo for the first time, learners use certain angles such as ‘rotate 120’ or ‘rotate 90’ to draw equilateral triangles or rectangles; they then compound these basic figures to represent complicate figure somewhat abstractly. Similarly using turtle blocks as basic form, we represent objects as 3D-figures similarly involves representation centered on an object’s characteristic form; the representation of simplified forms of objects with turtle blocks makes turtle block commands as easy to use as natural language.

This study uses turtle objects and turtle metaphor to propose a representation system in which the turtle creates various 3D-objects. In turtle block metaphor, the turtle creates a block that surrounds its periphery whenever it moves. That is, within the same level, we can order the turtle to construct 3D-object by using s (step forward), L, R (90-degree turn to the left, right). Symbolic commands are needed for the turtle to go upstairs or downstairs in 3D-space, and we have used symbols of u (up one level) and d (down one level) for the turtle to make turtle blocks while moving up or down as if riding an elevator. The object as seen in Figure 4 is a tetracube which is belongs to the mathematics curriculum in Korean elementary mathematics education, and the right-hand side object is created connecting the basic tetracubes.



do s s R s L s

→



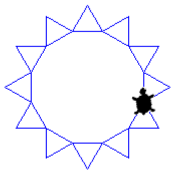
```
repeat 4 {
    do s s R s L s
    do R
}
```

Figure 4. A tetracubes and turtle block commands

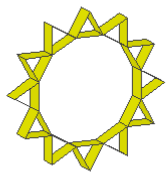
2.2. Turtle strap

Students who come in contact with various activities in Logo environment gain full experience of horizontal movement on a plane. Consequently, students need 3D representation that will provide experience of vertical, up and down movement. The turtle strap is perceived as a long paper strap that is folded here and there to create a figure. For this paper folding metaphor, the turtle uses the symbols m (move forward while making a band) and <, > (fold band towards left, right). For example, the right-hand side turtle strap in figure 5 is a 3D version of 2D-logo object by folding turtle strap at 30-degree angles, shown with the commands.

do m : make a unit turtle strap do > : folding up to right do < : folding up to left ddv : default degree value in folding turtle strap ※ In JavaMAL, '**rt**' means **'rotate'** (turn counterclockwise)**,** not **'**right turn'.



```
repeat 12 {
   repeat 3 { fd 10; rt -120;}
   fd 10; rt 30;
}
```



```
ddv = 30;
repeat 12 {
   repeat 3 { do m >>>> ;  }
   do  m < ;
}
```

Figure 5. 2D Logo commands and turtle strap commands

Note that the turtle strap is just a translation of a 2D-logo figure into a 3D-figure, and the turtle strap is a flexible concrete manipulative that can makes possible the activity of actually folding and unfolding a paper strap. As an application, Figure 6 gives an example how to represent physical rolling bridge in terms of turtle strap and turtle folding operation.

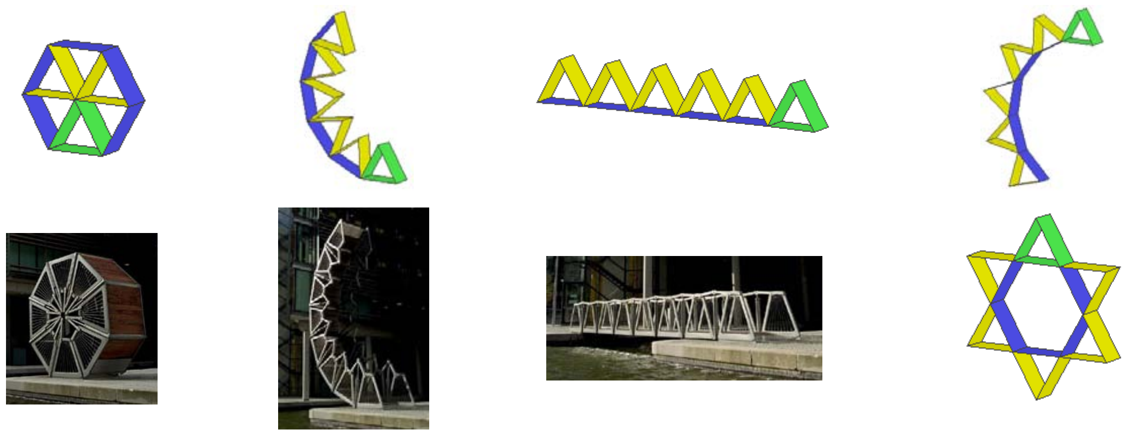
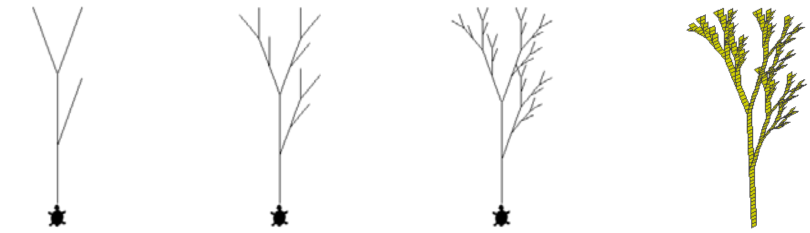


Figure 6. Rolling bridge and its turtle strap representation

2.3. L-system

Recursive functions play an important role in deductive and inductive reasoning and are widely used to explain natural phenomena such as fractals in 2D Logo. But Recursive functions are usually taught in secondary or university computer science curriculum because they take a difficult form of citing themselves. Proposed by biologist Lindenmayer, L-system is a formal representation system of diagrammatizing plant growth given by initial value and production rules. Wagon (1991) introduces turtle commands into this L-system to propose an L-system composed of turtle action without using difficult recursive functions. Following the same idea, we have defined f as the symbol for the turtle's forward movement of a certain distance, < and > as symbols for left, right turn of certain angles, and [and] as the symbols for saving and citing the turtle position and direction. Figure 7 shows production rules and tree growing image in each step; in reality, the trees in Figure 7 double in size as they move from left to right, but the image has been adjusted to identical height in order to compare the shape of the trees.

Production rule : X=' f [> X] f [< X] > X ' and f = ' ff '



Do_2 X; do_3 X; do_4 X; do_5 X; with turtle strap

Figure 7.Trees growing and turtle step representation

3. Mathematical objects and mathematization

3.1. Mathematical objects

Various objects, such as those in Figure 8, can be made using the turtle objects introduced above. The important point here is that an environment has been designed for the natural use of commands, like natural language, regarding the basic elements of representing solids. Such command is used in the same context of existing Logo, and several representation systems can be shared or used in combination. For example, turtle block can be used to make turtle shape in Figure 8.a. As in Figure 8.b, they can be used to understand the equation 1+3+5+7+9=25, visualizing the sum of odd numbers becomes a perfect square. Figure 8.c uses both L-system and Turtle strap at once. Here, each tree in Figure 7 is presented in the form of mathematical bar graph; the horizontal strap presents the domain of function while each tree presents a function value.

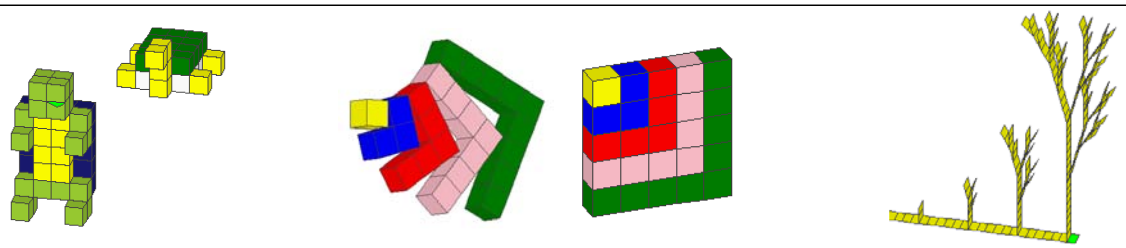
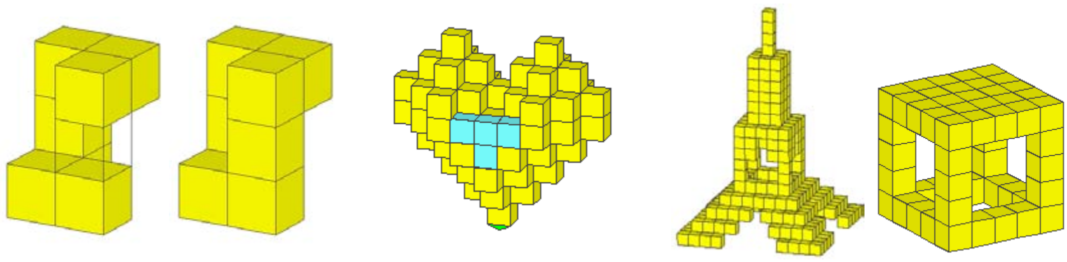


Figure 8.a Figure 8.b Figure 8.c

Figure 8. Mathematical objects

Turtle blocks can assist in the formation of mathematical objects for the learning of discrete mathematics-related content. For example, in the case of Euler characteristic number (v-e+f), all 3D-figures in South Korean curriculum are limited to cases in which the Euler characteristic number is 2. The Euler characteristic number can become a powerful investigation tool for various 3D turtle blocks as shown in the Figure 9. As the turtle blocks can provide various Euler characteristic numbers not equal to 2, the turtle blocks become appropriate thinkable objects for mathematization activities regarding Euler characteristic numbers. The figures shown in Figure 9.a is made by attaching 8 turtle blocks. The Euler characteristic number for this figure is 1 (Figure 9.a, left, a portion of the figure's center is made invisible). And the Euler characteristic number for the 3D heart (Figure 9.b) is 2. How about the Eiffel tower (Figure 9.c, left)? The answer is -6. The Euler characteristic numbers of two objects in Figure 9.c are the same. In fact, two objects in Figure 9.c are homeomorphic.



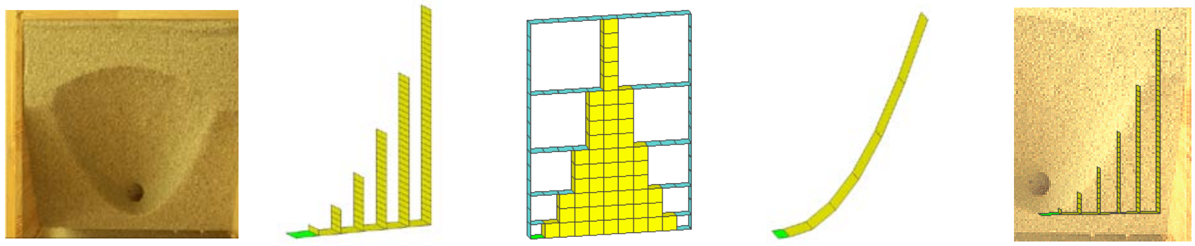
Euler #	1	2	?
	Figure 9.a	Figure 9.b	Figure 9.c

Figure 9. Turtle blocks and Euler characteristic numbers

3.2. Mathematization

Good manipulatives are those that aid students in building, strengthening, and connecting various representations of mathematical ideas (Clements, 1999). Because Turtle strap is a 3D-object, it is possible to do rotating or folding operation, and this enables the viewing of the object's movement from various side (for example, examination of only the x-axis or y-axis changes) or the examination of figure-to-figure transformation process through folding animation. L-system provides recursive behavioral patterns according to generating principles. Instead of focusing on turtle behavior, these patterns highlight the relationship between a figure's parts to its whole. Although the meaning found from turtle behavior had begun recursive thinking, the focus eventually lies in the translation of behavior into symbols and the finding of regularity between symbols. Focusing neither on drawing order nor turtle behavior but on the generating principles and change of symbols corresponds to a raise in standard.

Such activities are needed for mathematization as construct mathematics from the realistic phenomena. Figure 10 is an example of drawing mathematical concepts out of various phenomena, combining them and facilitating deep understanding of the mathematical concept through the learning by designing. Figure 10.a shows the physical parabola that would appear when making a hole in the sand box, and Figure 10.e shows a mathematical parabola that is constructed using the function equation $y=x^2$ since Figure 10.b represents a bar graph for the function $y=x^2$. Counting the blocks in Figure 10.c can be interpreted as an integration of the parabola and Figure 10.d represent a turtle strap constructed by attaching tangent segment of the parabola.



Physical parabola	Function equation	Integration	Differentiation	Mathematical parabola
Figure 10.a	Figure 10.b	Figure 10.c	Figure 10.d	Figure 10.e

Figure 10. Physical parabola and various mathematization of the parabola

4. Design of Communication Environment

Since our proposed representation system is based on action symbols, it is possible to combine them to represent various mathematical situations, and the representation can be saved, modified, and transmitted to others in web-board format in JavaMAL microworld. The activity of writing, revising, and transmitting ideas through web-based microworld is fundamental to create a web-based communication environment for mathematics education. Connections with the Learner Response System also enable immediate observation of a learner's present learning condition in an experimental environment. Going beyond previous levels of simple questioning over the exactness of knowledge, one can observe learning conditions in following microworld activity. The metaphor of Logo's turtle moving along with the mouse can also be used to create an electronic blackboard where learners exchange opinions on the microworld screen as a whiteboard.

4.1 Communication: Web-board system

Representing various media in a single space while designing their whiteboard, Hwang, Chen and Hsu (2006) introduce a function of representing and saving letters on a screen. The two methods of representing letters on the screen are input through a keyboard and drawing with a mouse. We have linked each of the two methods with turtle behavior. Input through a keyboard was linked to a turtle writing where he is, and letters drawn by a mouse were linked to a turtle following a mouse. When keeping in mind that the turtle draws extremely complicated drawings in Logo, the act of drawing letters while following a mouse becomes a very simple operation. When letters are drawn through a metaphor of 'turtle following mouse,' it becomes possible to represent this procedure with turtle behavior. That is to say, when this can be textualized and saved, and when this can be used to save and execute JavaMAL commands on a board, it becomes possible to communicate simple content as if writing on JavaMAL.

Text-based JavaMAL microworld representations can be easily linked with web-boards. Clements (1999) mentioned that benefits of computer environment are recording, replaying and linking between a concrete and symbol. The web-board in (http://www.javamath.com) is designed for linkage to JavaMAL microworld. As shown in Figure 11, the board is divided largely into two sections. On the left is JavaMAL microworld, and on the right is the board contained JavaMAL microworld commands and others replies. Commands are delivered between each section through JavaScript. When JavaMAL microworld commands and descriptions are written in the board, the board displays separate JavaMAL command buttons and a command screen. Because JavaMAL commands are displayed on that board in text format, it is unnecessary to save one's work, install programs, or saves files in order show others.

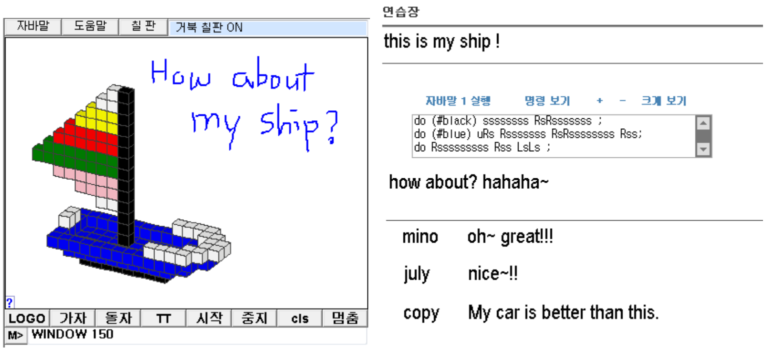


Figure 11. JavaMAL microworld and Web-board communication

4.2 Assessment: LRS (Learner Response System)

Comparing constructionism to constructivism, Ackermann (2004) mentions that the role of media is stronger in constructionism. Kafai and Resnick (1996) also mention constructionism as a mental construction made by physical construction; the physical construction in this case is constructed of a combination of several media, and Papert (1980) proposed Logo as an example of such media. There are many other studies aimed at introducing media in education. Hwang, Chen and Hsu (2006) present an environment that integrates various media through a whiteboard that they designed themselves, using it to study the influence that such an integrated environment has on the learning of a learner. The point of Hwang's white-board is that the learner represents opinions regarding a single topic by voice and writing. This presents a possibility that many students will express their opinions as well as receive other people's opinions and revise them. Studies have also been made on interactive response systems such as Clicker in order to assess the current learning conditions of students in the classroom scene using media.

Bruff (2009) mentions that the use of IRSs increase students' class concentration, class participation, and class enjoyment, having the effect of providing valuable feedback regarding the class to both instructor and learner. In particular, Roschelle (2003) discusses the role of IRSs in a network environment. This study integrates microworld to network-based media, such as computers, to construct a richer learning environment. LRS (Learner Response System) is a type of assessment tool used to measure current comprehension conditions of learners in class. The system operates in a similar manner to network-based IRSs such as WILDs mentioned by Roschelle (2003). The LRS introduced in this study was developed for use in conjunction to JavaMAL microworld. Here, learners connect to web-boards, connecting to LRS as if reading postings. They then experiment in the given JavaMAL environment and respond based on the experimentation. This is possible because JavaMAL microworld is a text-based environment. Through alteration in settings, LRS can also be changed into a testing environment for prepared questions. In Figure 12, on the left is the LRS screen for statistics, in the right is the LRS screen for question and in the middle is the JavaMAL microworld for experiment.

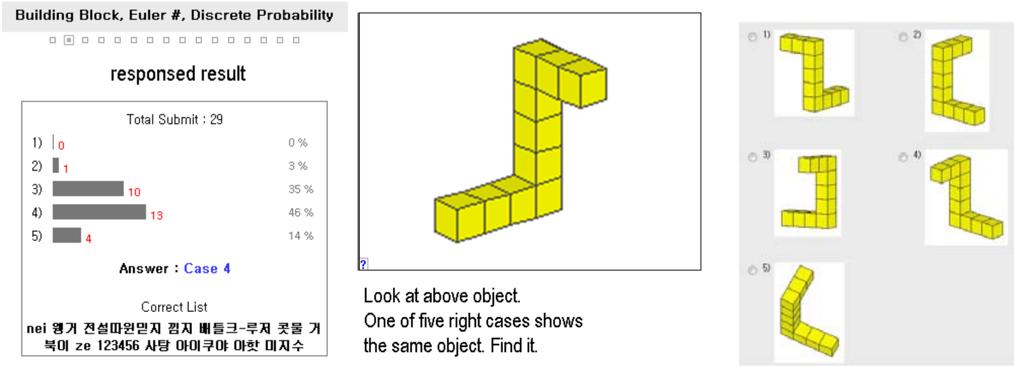


Figure 12. Framework of the LRS (Learner Response System)

5. Closing remarks

5.1. Turtle tiles and turtle nets

Cho, Han, Jin, Kim and Song (2004) showed that drawings made with Logo's turtle can be made into turtle tiles and given movement animation. Turtle tiles are dynamic objects that can be given rotating, moving, and flipping operations with a mouse. Whereas Logo draws fixed drawings, turtle tile changes fixed drawing into movable objects. Here, a metaphor of 'a turtle laying eggs' has been used. The Logo's turtle lays eggs while moving to draw pictures. And at the command to make turtle tiles, turtle tiles are made into the polygon created when the eggs are connected. Figure 13, left, is a base drawing that the turtle makes while hiding invisible eggs on the way. Figure 13, right, shows turtle tiles that are made by the hidden eggs and moved by the mouse or commands.

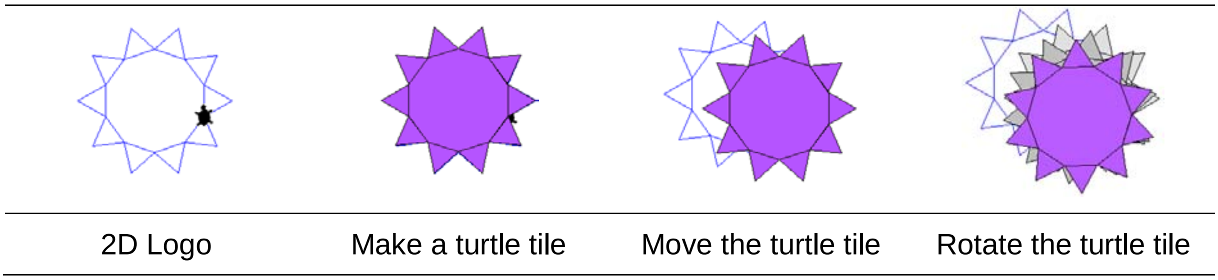


Figure 13. Turtle figure and turtle tile

3D-figures can be made by attaching several turtle tiles and folding them in controlled angles. The turtle moves while laying eggs even when making a development figure, and newly created turtle tiles attach to existing turtle tiles as turtle tiles are created by laying two eggs on top of two previously-laid eggs. In other words, in order to be added, a new turtle tile must share one side with an existing turtle tile. Figure 14 shows the order in which the turtle created and attached turtle tiles, along with the folding of the development figure.

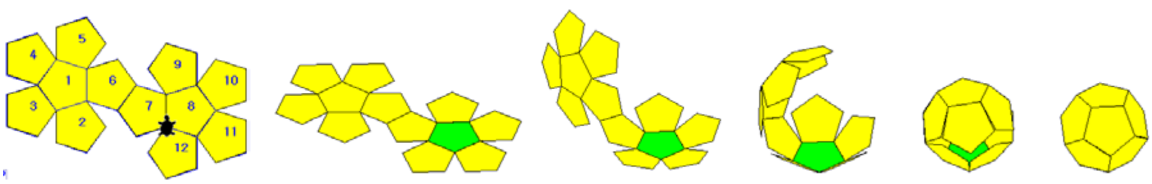


Figure 14. Turtle net and polygon folding

5.2. Turtle vector geometry

Abelson and diSessa (1980) call Logo as turtle geometry, comparing it to coordinate geometry while stating its characteristics as intrinsic, local, and procedural. However, depending on the situation, there are cases that require the turtle to have an absolute axis. Introducing the metaphor of 'turtle on flowing river,' Cho, Kim and Song (2006) show that concepts of basic calculus can be introduced to lower-grade students as pre-calculus. In the Logo proposed by Papert (1980), the turtle moves across a white field covered with snow. Now suppose that the turtle is on a river that flows from left to right at a regular speed, and the turtle is swimming upstream towards the opposite side of the river. Consequently, when the turtle stays still, it will flow along with the river; and when the turtle swims at a regular speed, it will move in a diagonal towards the upper right direction. This is the metaphor of 'turtle on flowing river.' As the most important core concept in Logo, action symbols are not merely symbols that represent turtle movement but are concepts that can develop and expand into various directions. Move command is a vector representation that combines move in horizontal and vertical directions. Whereas Logo's move commands of 'forward' and 'rotate' are relative, movement using 'move' becomes movement on an absolute axis.

Abelson and diSessa (1980) state that turtle geometry and vector geometry are two different representations of the same thing, and that two or more representations of the same thing can often lead to insights that are not inherent in either of the representations alone. Cho, Kim and Song (2007) propose a circle model for drawing cycloid-family through the composition of move command, along with two representation methods for illustrating this in DGS. Figure 15 shows figures drawn by move commands and trigonometric commands.

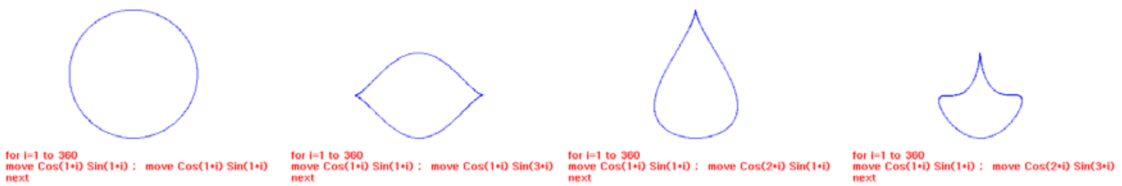


Figure 15. Figures with 'move' commands

5.3. Concluding

In this study, we design an educational Logo-based microworld environment equipped with 3D construction capability, 3D manipulation, and web-based communication. Extending the characteristics of 2D Logo's behavioral symbols, we design symbolic representation system that can create turtle strap, turtle blocks, do_n recursion. The proposed symbolic representations can be used for various mathematization activities, web-based collaborative learning, communication, and assessments, and we examine actual examples of design that connects LRS and web-board to JavaMAL microworld.

6. Acknowledgement

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7. References

Abelson, H. and diSessa, A. (1980) *Turtle geometry.* Cambridge, MA: MIT Press.

Ackerman, E. K. (2004) Constructing knowledge and transforming the world. In M. Tokoro and L. Steels(Eds.), *A learning zone of one's own: Sharing representations and flow in collaborative learning environments.* Amsterdam: IOS Press.

Bruff, D. (2009) *Teaching with classroom response systems: Creating active learning environments.* Jossey-Bass Publishing.

Cho, H., Han, H., Jin, M., Kim, H. and Song, M. (2004) Designing a microworld: activities and programs for gifted students and enhancing mathematical creativity, *Proceedings of ICME TSG 4*, 110-120.

Cho, H., Kim, H. and Song, M. (2006) The qualitative approach to the graphs of functions in a microworld, *The SNU Journal of Education Research* **15**, 129-140.

Cho, H., Kim, H. and Song, M. (2007) Mediating model between Logo and DGS for planar curves, *PME Conference 2007*.

Clements, D. (1999) 'Concrete' manipulatives, concrete Ideas, *Contemporary Issues in Early Childhood* **1(1)**, 45-60.

de Jong, T. and Monica, G. F. (1996) Types and qualities of knowledge, *Educational psychologist* **31(2)**, 105-113.

Eidswick, J. (1986) Cubelike puzzles – what are they and how do you solve them, *The American mathematical monthly* **93(3)**, 157-176.

Hwang, W. Y., Chen, N. S. and Hsu, R. L. (2006) Development and evaluation of multimedia whiteboard system for improving mathematical problem solving, *Computers and Education* **46(2)**, 105-121.

Kafai, Y. and Resnick, M. (1996) *Constructionism in practice.* NJ: Lawrence Erlbaum Associates, Publishers.

Klopfer, E., Colella, V. and Resnick, M. (2002) New paths on a StarLogo adventure, *Computers and Graphics* **26(4)**, 615-622.

Morgan, C. and Alshwaikh, J. (2008) Imag(in)ing three-dimensional movement with gesture: ''playing turtle'' or pointing?. *Proceedings of the British Society for Research into Learning Mathematics* **28(3)**.

Papert, S. (1980) *Mindstorms: children, computers, and powerful ideas.* Cambridge, Massachusetts: Perseus Publishing.

Roschelle, J. (2003) Unlocking the learning value of wireless mobile devices, *Journal of Computer Assisted Learning* **19(3)**, 260-272.

Wagon, S. (1991) *Mathematica in action*, NY: W. H. Freeman.

Wilensky, U. (1991) Abstract mediations on the concrete and concrete implications for mathematics education, In I. Harel and S. Papert(Eds.), *Constructionism*, 193-203. Norwood, NJ: Ablex.

van der Meij, J. and de Jong, T. (2006) Supporting students' learning with multiple representations in a dynamic simulation-based learning environment, *Learning and Instruction* **16**, 199-212.