Building understanding: Geometry for Design at the Community College of Philadelphia

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Abstract

Technology is radically altering the landscape, no more so than in the field of mathematics education. In particular, electronic learning resources lend themselves easily to a constructionist approach to teaching and learning mathematics. Along with traditional hands-on, activity-based methods, technology can help create productive environments for studying geometry. This paper compares outcomes for community college students studying two different college math courses, Geometry for Design and Intermediate Algebra. Students enrolled in visual design curricula -- CAD (computer assisted design), construction management, fine/applied art and interior design -- can study Geometry for Design, a technology-rich course that uses model-making, interaction and hands-on activities, along with traditional methods.

Geometry for Design students' achievement was compared with the achievement of students in a traditional lecture-based college algebra course that generally does not use classroom-based technology or hands-on activities. Geometry for Design students seemed to have a higher rate of success than students studying Intermediate Algebra. Geometry for Design students also seemed to hold more strongly positive opinions about hands-on methods for learning mathematics. Geometry for Design students were more likely to strongly agree that they had acquired strategies for thinking about problem-solving and that they had developed a better understanding of how math relates to the real world.

Figure 1. Students constructing models of Platonic Solids

Keywords
constructionism design education geometry mathematics technology

Introduction

This paper compares the achievement of community college students in a technology-rich geometry course with the achievement of students in a traditional lecture-based algebra course. Geometry for Design (Math 137) is an interdisciplinary, technology–enhanced geometry course that is taken primarily by students in the College's Construction Technology, Computer Assisted Design (CAD), Art and Interior Design programs. Intermediate Algebra (Math 118) is a traditional lecture-based course with no designated classroom technology component. Many students register for Intermediate Algebra because it is a prerequisite for most curricula at the Community College of Philadelphia. Outcomes for students in Geometry for Design courses were compared with outcomes for students in Intermediate Algebra courses. Geometry for Design students' attitudes towards learning geometry from a constructionist perspective were explored by questionnaires. Geometry for Design students seemed to have a higher rate of success than students studying Intermediate Algebra. Geometry for Design students also seemed to hold more strongly positive opinions about hands-on methods for learning mathematics. Geometry for Design students were much more likely to strongly agree that they had acquired strategies for thinking about problem-solving and that they had developed a better understanding of how math relates to the real world.

Geometry for Design at Community College of Philadelphia

More than half a century ago, in "Le Modulor", Charles Edouard Jeanneret wrote:

Passée la porte des miracles …Mathematics is the majestic structure conceived by man to grant him comprehension of the universe. It holds both the absolute and the infinite, the understandable and the forever elusive. It has walls before which one may pace up and down without result; sometimes there is a door: one opens it --- enters--- one is in another realm, the realm of the gods, the room which holds the key to the great systems. These doors are the doors of the miracles.

More than half a century later, technological marvels that we take for granted—computers, air travel, ubiquitous cell-phones, medical imaging technology, and perhaps most interesting to Le Corbusier, tall buildings, to name but a few—would have been impossible to realize without mathematics. Excitingly, new technologies utilizing principles of visual design are emerging, creating jobs in architecture, animation, construction, engineering, applied art and interior design. These rewarding careers require strong mathematical skills and the ability to frame and solve quantitative problems. A solid foundation in geometry is essential for the development of problem-solving ability, and Geometry for Design is an important course for Community College of Philadelphia students in visual design and construction curricula.

The unfortunate reality is that, at many colleges today, students often begin their study of visual design without a strong knowledge of geometry. This impedes progress in many ways: students may be unable to make simple area calculations or conceptualize nets of three-dimensional solids. To address this problem, faculty from Community College of Philadelphia's Mathematics, Art, Architecture and Construction Technology departments created a new math course, Geometry for Design. Course development was supported by funding from the U. S. National Science Foundation and Geometry for Design was first offered 1998.

One characteristic of geometry is its immediacy. In contrast to other branches of mathematical study that rely heavily on abstraction, the study of geometry can begin with commonly observed and experienced phenomena from the physical world. Accordingly, the pedagogy of experience-based learning was adopted, and accordingly lecture lesson-plans include hands-on activities and computer applications and investigations. Geometry for Design combines a sound preparation in the basic concepts and techniques of plane and solid geometry with a thorough exploration of geometry's concrete applications in architecture, construction, art and design. Michael Serra's, "Discovering geometry: an investigative approach v. 4" is the assigned text and Geometer's Sketchpad software is used in the computer classrooms. Topics include: traditional straightedge-and-compass and computer-based construction methods; properties of triangles, polygons and circles; plane transformations, symmetry and tessellations of two-dimensional figures; area; 3-dimensional polyhedra; volume; the Pythagorean Theorem; ratio and proportion and similarity. Some study of formal proofs is included. Topics from art and architecture are studied, including perspective drawing, the design of arches and domes and an exploration of the Golden Mean and its relevance to architecture, art and science. Students create portfolios of manual and computer-based drawings and build 3-dimensional models. Classes meet in a computer-equipped classroom, and Geometer's Sketchpad is used extensively to explore ideas, make conjectures, literally draw conclusions and formulate proofs. Students also use Internet web sites that relate to geometry, architecture, construction, art and design.

Geometry for Design incorporates "hands-on" group activities to get students involved in "building knowledge structures". For example, the course begins with an ice-breaker "Building Blocks" puzzle that has multiple solutions requiring good visualization skills. Students are also introduced to the idea of working together in groups, which are used throughout the course.

Early on in the course students are exposed to the concept of inductive thinking and formula development. Given a set of regular polygons, the task is to try to find the number of diagonals of an 18-gon that has a somewhat daunting-looking diagram resembling "String Art" gone mad (Seymour and Beardslee, p. 42).

In the process of constructing the four points of concurrency of a triangle (incenter, circumcenter, orthocenter, and centroid) students practise measuring , observing and drawing using traditional compass-and- straightedge techniques. They also master the modern reincarnations of these time-honoured skills by recreating these constructions in the computer-based environment of Geometer's Sketchpad. These skills, evinced by the illustrations and drawing exercises in Lawlor's "Sacred Geometry " , are indispensable for students of design who will use geometry in the real world.

Much hands-on work is done on the properties of circles. In particular, students learn about pi as the ratio of circumference to diameter of a circle by rolling out the circumference of a wheeled object (e.g. a trolley or a bicycle tire) , measuring its diameter and thus discovering the approximate value of pi. This activity relates well to real- world applications such as oodometers and pedometers. The derivation of the area of a circle is explored by juxtaposing 12 sectors into a "rectangle" and positing its area as (radius) x (½ the circumference).

The study of isometry transformations -- translation, reflection and rotation-- is an especially fertile area for the use of hands-on activities using computer software. Students can relate to these topics, as they are used extensively in computer assisted design and graphic design. Students exercise their creativity by building their own tessellations using translation and rotation. Web sites and videos introduce students to geometrical aspects of the work of the artist M.C. Escher.

While exploring Pythagoras's Theorem, students use strings of holiday lights to investigate the condition for a right-angled triangle .Three students are designated as the "vertices" while a fourth is charged to count the number of "units" on each side and verify the right angle. This activity is a variation of the knotted ropes techniques used by Ancient Egyptians. Four different proofs of Pythagoras's Theorem are presented, including Bhaskara's proof, Leonardo da Vinci's hexagonal proof and President Garfield's trapezoidal proof. Students use a hands-on Geometer's Sketchpad computer activity to explore a "juxtaposing areas" proof of the Theorem.

Figure 2. Students exploring Pythagoras's Theorem

Rectangular solids and their volumes are explored and formulae are derived using small 1cm3 blocks. A set of small hollow translucent "Power Solids" can be filled with water to demonstrate that the volume of a cone is one third of the volume of a cylinder of the same radius and height. The formula for the volume of a pyramid is explored in similar fashion and students derive the formula for the volume of a sphere by comparing its two hemispheres' volume to the volume of an associated cylinder. This demonstration can be a bit messy with liquid being poured from solid to solid but students enjoy this hands-on activity and it is well worth doing. Students explore and discover the Euler formula $V+F-E=2$ (Seymour and Beardslee, p. 33) using cocktail sticks and mini-marshmallows (fig. 1) to make models of polyhedra. Using nets, students construct and creatively decorate cardboard models of the five Platonic Solids, including the famous "cheese-a-hedron".

Figure 3. Students' polyhedral models

Students access on-line web sites and they state that they really enjoy using artist/mathematician George Hart's web site georgehart.com to learn about complex polyhedra in the context of art/architecture history.

When studying similarity, students use traditional compass and straightedge constructions and computer-generated activities to investigate similarity criteria (AAA, SSS, SAS, ASA) for triangles. Students employ similar triangles to indirectly measure the height of a tall object (e.g. a ceiling sprinkler head) using a plane mirror on the ground between the sprinkler and the observer.

Finally, students explore the Golden Mean and its significance for geometry, design, biology, art and architecture. Golden rectangles are constructed using a compass and straightedge and students observe real word objects that are configured in golden rectangular proportions, such as credit cards and ID cards, packs of cigarettes, many containers of different products and the ubiquitous iPod.

To assess student achievement in the Geometry for Design course, seven computational homework quizzes, a midterm exam and a final exam, two short written papers and a portfolio of computer-generated drawings are used. Grades for the course are awarded as follows: A = 90-100%, B = 80-89% , C= 70-79 % , D= 60-69%, F= 0 - 59%.

Intermediate Algebra at Community College of Philadelphia

Most students at Community College of Philadelphia study Intermediate Algebra at some time during their college career. Its popularity is a result of compulsion, not inclination, as it is a prerequisite for admission to most degree and certificate programs, especially those in the fields of engineering, science and technology. As is the case with other college mathematics courses, Intermediate Algebra functions as a filter or "gatekeeper" course. Topics include the Real number system, systems of linear equations and inequalities, polynomials, rational expressions, radical expressions, and quadratic equations. Most instructors use a customized intermediate algebra text and students are usually taught in a traditional classroom using a "chalk-and talk" lecture-based format. Generally, students do not have access to classroom computing facilities, although there is a Math Learning Lab which provides some computer support. There is no departmental consensus on the use of technology in the classroom: some instructors encourage the use of calculators and computers, while others discourage and even prohibit their use. Meanwhile, students have embraced the twenty-first century digital age and want to use graphics calculators, computers and even cell phones for graphing and computation.

Assessment of student achievement in the Intermediate Algebra courses taught by the author is measured by seven computational homework quizzes, midterm and final exams. Grades for the course are awarded as follows: A = 90-100%, B = 80-89% , C= 70-79 % , D= 60-69%, F= 0 - 59%.

Comparing outcomes and opinions

To compare the achievement of students in a technology-rich geometry course with the achievement of students in a traditional lecture-based algebra course, records of students' final grades were analyzed. Data was obtained from five sections of Geometry for Design and nine sections of Intermediate Algebra taught by the author from Fall 2007 through Fall 2009. If "success" is earning an A, B or C grade in a course, the Geometry for Design students: chance of success was 95/135 $\approx$ 70 %, while for Intermediate Algebra students , the chance of success was 173/288 ≈ 60 % and so there was a small but statistically significant difference between these proportions. Geometry for Design students had a slightly better chance of earning an A, B or C than Intermediate Algebra students.

In addition, there seemed to be a qualitative difference in the students' opinions about the two mathematics courses. Short exit questionnaires with Linkert scale response options were used. When asked at the outset whether they found mathematics interesting and enjoyable, there was very little difference between the two groups' responses, with some strongly agreeing, some strongly disagreeing and some with neutral opinions. However, there was a discernable difference in the two groups' responses to questions that asked about their attitudes towards "hands-on approaches" to learning mathematics, towards understanding how mathematics relates to the real world and towards developing strategies for thinking about word-problems. Generally, Geometry for Design students strongly agreed that they liked hands-on approaches, strongly agreed that they understood how math relates to the real world and strongly agreed that they had developed strategies for problem-solving through the study of geometry. The Intermediate Algebra students generally agreed with the statements, but were much less likely to strongly agree. Furthermore, many of the Geometry for Design students wrote unsolicited positive comments about the course but hardly any of the Intermediate Algebra students wrote anything about their experience of learning algebra. For example, Geometry for Design students wrote:

"I really am not a math person but I did enjoy this course, more than any other math course I have taken," and " I had fun", and " Wow, a tolerable math class".

This is a statement by Chin, who studied Geometry for Design and transferred to art college:

"I'm thinking more about how technology is (sic) made an impact on the art world. Today I was reading an article in the City Paper. It was about a guy who's currently the Multimedia Director at University of the Arts. He's used the technology to create work that has the same vision and insight as traditional styles of art. I think of the geometry class and how we integrated the computer along with the traditional methods of geometry to come up with a more in depth idea of how the world and geometry work together. I think the time is near where the technology becomes the standard by which we communicate ideas and information on a daily basis."

Figure 4. Students exploring Euler's Theorem

Another statement -- from Jennifer, an Art major who found success with Geometry for Design after failing Intermediate Algebra three times in a row -- reveals the exasperation many students experience with learning algebra in a traditional classroom environment:

"…I kept dropping (Intermediate Algebra Math 118) too late past the time when you could drop. I was receiving 'F's and this kept driving me to keep taking the class to turn the 'F's into an A. …But I was unsuccessful. Then I went to register for this semester with (Head of Art) when she told me that I needed to take a math to graduate in May . I almost cried. She suggested I take Geometry for Design and told me to give up on 118 after reviewing my transcript. I was really hesitant and nervous, but then I talked to other students who had taken this class last semester, and they gave it good reviews. I really enjoy this class. I don't even consider it a math class. I can actually say that I look forward to going to this class. I understand what I'm doing because I can see how everything that is taught relates to the world. I always liked geometry better than algebra; I was always interested in shapes and design….I loved that movie… on MC Escher…"

Thus, after a series of F's in Intermediate Algebra Math 118, Jennifer earned an A in Geometry for Design, finally graduated from Community College of Philadelphia, and also won a $10,000 college scholarship. Although Geometry for Design Students seemed much more enthusiastic about learning mathematics, future study is indicated to better understand the differences in students' attitudes towards learning geometry from a constructionist perspective.

Conclusion

Students who studied Geometry for Design seem to succeed at a higher rate than students who studied Intermediate Algebra. Students who have studied Geometry for Design volunteered that it was a relevant and even at times enjoyable experience; Intermediate Algebra students rarely seemed to respond so favourably. Further study is required before drawing any broader conclusions. Future studies might compare the achievement of students taught geometry from a constructionist perspective using technology and hands-on activities with the achievement of students taught geometry using traditional methods with no access to technology and hands-on activities.

Studying Intermediate Algebra can be a frustrating, potentially unsuccessful and generally terminal mathematical experience for many students. However, Geometry for Design students are immersed in contexts of art, architecture, construction and interior design, and seem to benefit from a geometry course that employs a constructionist approach with technology enhancement and hands-on, activity-based methods.

Echoing Le Corbusier, a Geometry for Design student wrote, "I think this course is very helpful with my understanding of the world around me since mathematics is the true language of the universe…". On a parallel tack, Schneider (p. xxvii) quotes Scottish zoologist and classical scholar Sir D'Arcy Wentworth Thompson who wrote," The harmony of the world is made manifest in Form and Number, and the heart and soul and all the poetry of Natural Philosophy are embodied in the concept of mathematical beauty.". Perhaps it is the nature of a constructionist approach to learning geometry that resonates for students in visual design curricula. More research into this question is indicated.

Thus, as they explore the fundamentals of geometry, students in visual design curricula are finding that they have a better chance of success by studying Geometry for Design, where they are learning some interesting, useful and powerful mathematics.

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