

# Visualization processes in a 3d tool designed for engineering activities

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## Abstract

The development of spatial and visualization abilities in engineering education has been considered as a process unrelated to the use of external representations as they are usually “monitored” through tests meant to define the level of accuracy in which the students create, store, recall from their memory and manipulate internal mental images. Professional computational environments, when employed, make visualization processes even more obscure for engineering students, as the boxed up mathematics they include restrain the potential for genuine explorations inside the 3d space.

Constructionist environments designed to offer features that support dynamic visualization inside the 3d space, on the other hand, provide a richer context for engineering students, as they seem to allow them to utilize and develop elements of their spatial and visualization abilities. Half-baked microworlds developed in such environments may challenge students to rebuild artefacts according to their own conceptualizations of the 3d space, while Logo programming components could be used to shed light to procedures that they appear black boxed in professional software.

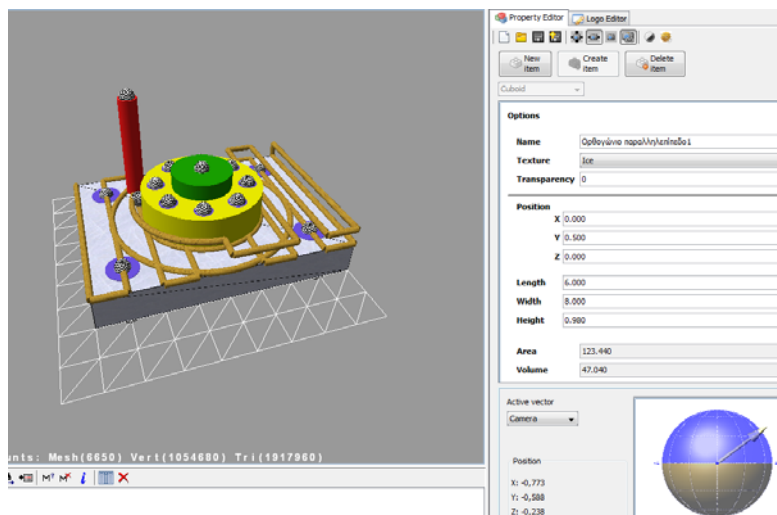


Figure 1. Representing an engineering component in a 3d space and creating Logo programs to define the cutting tool's path

## Keywords

Spatial & visualization abilities, engineering education, constructionism, dynamic visualization

*This work among others led to a European Commission project on collective reflection of collaboration in constructionist environments called “Learning to learn together: A visual language for social orchestration of educational activities”. FP7-ICT-2009-5, Technology-enhanced Learning, N. 257872.*

## Spatial and visualization abilities in mathematics and engineering education

Spatial and visualization abilities have long been considered as being connected to mathematical learning and aspects of it, such as geometrical thinking and problem solving (Bishop, 1980; Presmeg, 2008; Clements and Battista, 1992). Apart from mathematics education, however, the development and improvement of spatial and visualization abilities have also been recognised as fundamental for the engineering students' education (Sorby and Baartmans, 2000). The main argument for this thesis is that the engineers through visualization bring forth the ideas they have in mind -using in most cases engineering drawings as a vehicle- and thus produce creative solutions to the design and technical problems they encounter (Ferguson, 1992). Therefore, several tertiary institutions urge engineering freshmen to take Spatial Ability Tests, so as to define if they possess these abilities and their level of elaboration, and then to get retested once again after attending "remedial" courses that are thought to accentuate those skills (Potter and Van Der Merwe, 2003). The perception of visualization from a cognitive psychology point of view as the process of producing, retrieving from memory and manipulating mental images in one's mind, with the minimal interference of external representations -including technology (for a discussion see Gutierrez, 1996)- has led to the development of various spatial abilities and mental rotation paper-and-pencil tests, that are designed to determine the level of accuracy and speed in which engineering students may create and manipulate internal representations of complex 3d objects, often unfamiliar to engineers. The use of computer environments, where available, has been mainly viewed as new -alternative to the paper and pencil- means for the students' spatial instruction through drawing and sketching and has been restricted to the use of CAD packages (Leopold et al., 2001). The question that raises here is what the mathematics education's view of visualization and the constructionist paradigm has to offer engineering education.

Although mathematics educators often use diverse definitions to explain terms such as spatial and visualization abilities, there is no doubt that they hold a long-standing research tradition in this specific field. Taking a different strand than the one presented before in the engineering education's case, visualization has been considered by mathematics educators as the ability to represent, transform, generate, communicate, document, and reflect on visual information (Hershkowitz, 1989, p. 75). The development of this ability, however, has not been regarded as independent of the use of external representations and especially technological artefacts (Gutierrez, 1996; Zimmerman & Cunningham, 1991). Particularly, computational environments that hold the potential for the creation and manipulation of dynamic images (dynamic visualization), may contribute to the development of the students' spatial and visualization abilities (Christou et al., 2007), as these often appear to be interwoven with the software's semantics and functionalities (Kynigos & Latsi, 2007). This kind of environments may not only empower students to build and observe inside them structures of 3d objects with certain properties, but also to transform those constructions in real time (Arcavi & Hadas, 2000), through dynamic manipulation. Real-time dynamic manipulation of 3d constructions could potentially enhance students' conceptualisation of the 3d space and enable them to work inside a three-dimensional frame of reference with the use of x, y, z Cartesian coordinates (Kynigos et al., 2007). Logo components in such computational environments that allow the manipulation of symbolic code -generating graphical effects on the screen- may also contribute to the students' mathematically driven navigation and orientation inside the 3d space, as they come to build bridges between the intrinsic turtle geometry and the Cartesian geometry (Kynigos, 1991).

Looking once again at the engineering education context, one may say that constructionism, as an idea, contains inside it what engineering students at the school labs usually do. They construct their knowledge about the engineering world as they collaboratively design, calculate, measure, program, shape and cut meaningful for them external artefacts. Tearing down

something that they don't know how it works, just to explore its functionalities or repair it if broken and show it to others is regarded as a process belonging to their established habits of mind (Cuoco et al., 1996). However, when it comes to the use professional computational environments in which 3d objects can be designed and represented -such as CAD packages- engineers often find themselves to perform routine operations, such as pressing buttons on the environment's UI, unable to take them to pieces and find out what lies beneath. Thus, procedures that seem to be connected with visualization in the 3d space, such as changing a 3d object's scale or rotating it around an axis, remain boxed up when using professional computational environments, as their attainment is just a matter of selecting and clicking on the correct button (Kent & Noss, 2001). As mathematics become invisible and crystallized in black boxes inside sophisticated 3d CAD environments, visualization in the 3d space becomes a more and more obscure process for the engineering students.

Bringing mathematics educators experience in developing computational environments meant for mathematically driven visualization and adopting the constructionist perspective, we designed and developed a computational environment called MachineLab Turtleworlds (MaLT). The dynamic visualization of the 3d objects represented inside MaLT's 3d virtual space is achieved both through their direct, real-time manipulation and the execution of Logo procedures that define their properties and behaviours. The 3d manipulation of a camera's position and direction inside the 3d virtual space allows observation from multiple viewpoints, giving dynamic visualization a new powerful tool. The "3d Modelling & Cutting" microworld we developed in MaLT was specifically designed to challenge engineering students' conceptualisations of the 3d space as they attempt to represent inside it mechanical engineering components and generate for them shaping and cutting procedures using Logo programming. In this paper we report on a small-scale research conducted to study high school engineering students' visualization processes as they work with the "3d Modelling & Cutting" microworld. Analysing their reasoning activity while they explore, build and manipulate their constructions in the 3d space, we attempt to identify the kinds of spatial and visualization abilities they employ and monitor their development as the students interact with the computational environment.

## The computational environment

### The MachineLab Turtleworlds (MaLT) environment

MaLT is a programmable environment that allows the creation, exploration and dynamic manipulation of 3d geometrical objects, graphically represented inside a 3d virtual space. The objects visualized inside it are either *constructed* by the environment's Turtle, when running Logo procedures and commands, or *inserted* by the user, after selecting them from a library that contains numerous ready-made stereometric objects, such as cuboids, cylinders and cones. Inheriting elements from "E-Slate 2d Turtleworlds", MaLT integrates symbolic notation -in the form of Logo programs- with the dynamic manipulation of the 3d geometrical objects through the use of specially designed Variation Tools (Kynigos & Psycharis, 2003).

To observe her/his constructions inside the 3d space from different viewpoints, the user has at her/his disposition three different cameras: the Floor View, the Side View and the Main/Front View camera. However, building and manipulating geometrical objects in MaLT, is not restricted in solely looking at the 3d world from static 2d orthographic views. A 3d Camera Controller gives students the opportunity to navigate around, inside and through their constructions, offering the potential for new ways of visualizing the 3d space and the geometrical constructions inside it. The dynamic manipulation of the objects themselves in the 3d virtual space, along with the dynamic manipulation of the camera, can prove to be powerful tools for understanding mathematical concepts in the 3d space, and a resource for solving mathematical problems that require the use of spatial information.

### The “3d Modelling & Cutting” microworld

Using MaLT as a platform, we built the “3d Modelling & Cutting” microworld in which the Turtle is replaced by a cylinder representing a milling machine’s cutting tool (Figure 1 and Figure 2).

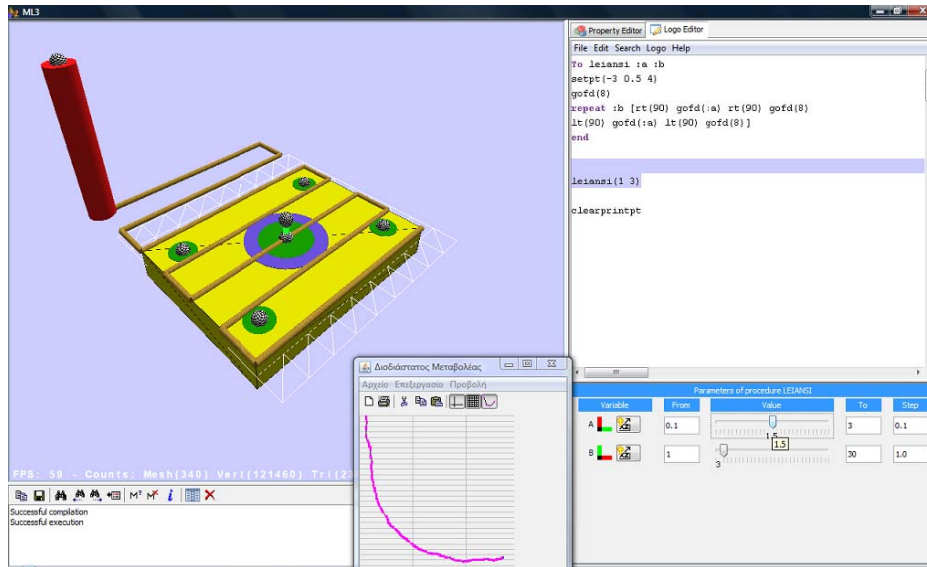


Figure 2. Planing an object’s XZ surface using a milling machine’s tool

The tool can be programmed through Logo commands to move inside the 3d space and perform in it several machining procedures (such as drilling and planing), leaving on its way a trail behind. This trail, when generated through the use of Logo procedures that encompass variables, transforms from static to dynamic and becomes connected to the environment’s Variation Tools. Manipulating the Variation Tools by performing dragging actions (Figure 2), the user attributes each time new values to the procedure’s variables, causing the tool’s path to change its direction, length, and/or position in the 3d space, which allows the parametric programming of several machining procedures.

## Research design and methodology

Our research approach was based on the idea of studying learning in authentic settings through “design experiments” (Cobb et al. 2003). “Design experiments” aim to contribute to the development of grounded theories on “how learning works” and are conducted with the intention to shed light on the relationships between the material designed for the experiment (usually innovative technological artefacts having added pedagogical value) and the learning processes within a specific context of implementation.

### Context and participants

The experiment took place in a Secondary Vocational Education school in Elefsina –an industrial town near Athens- with three 12<sup>th</sup> grade students, studying mechanical engineering and having a particular specialization in Programming Computer Numerical Control (CNC) Machines. As part of their two-year tuition, these students had taken courses in working with Computer Aided Design and Computer Aided Manufacturing environments and had operated real CNC milling and lathe machines with the help of their lab teacher. All of them had also been working at the time of the research at middle-scale mechanical engineering workplaces as inexperienced workers. Their school, as well as their workplace training, had given them several experiences in working with Cartesian coordinates, mostly, however, in two dimensions.

The experimentation process was held at the school's CNC lab for 15 school hours, with the students working together as one group. All three shared one PC, while they had at their disposal the computational environment's manual and a notebook for their ideas, remarks, sketches and drawings. Adopting a "participant observation" methodology, the researcher, who also was a Mechanical Engineering teacher, did not intervene to give out instructions or to provide the "correct answer", but chose to pose meaningful -often intriguing- questions at certain time points, so as to encourage students to continue their explorations, collaborate, share and discuss their ideas with each other.

### Tools and Tasks

Drawing on the idea of "layered learning design" (Kahn et al., 2006), we divided the activity sequence in three distinct phases, the first of which also served as a "familiarization with the computational environment" phase. In order to recreate a situation that could be experientially real for the students (Gravemeijer et al., 2000) and close to their professional life, we decided not to provide them any information regarding the features of the environment and ask them to work with it as if they were in their workplace and all three as colleagues had to explore and understand its functionalities, so as to represent in it objects that would be consequently cut in CNC machines. The only available help was coming from the environment's manual -with not much revealed about the 3d virtual workspace- challenging students to figure it out for themselves, if to work in this environment.

#### *Phase 1: An unfamiliar 3d space*

As we had decided not to provide the students any instructions regarding the environment's functionalities, we chose to develop for the first phase of our experimentations a relatively simple microworld. This microworld consisted of just one ready-made stereometric object, a rectangular parallelepiped sized 5x5x1, which, with the Top View being activated on the environment, looked like a 2d parallelogram sized 5x5. Working mostly with 2d environments, the students usually recognise as the "Width" dimension the one represented in the Y axis and the Y axis as being the vertical one on the surface of their screen. To induce the students to explore the computational environment's virtual space in all three dimensions, we decided to ask them to resize the object to 3x4x1 (length x width x height) and place it at the (0, 0, 0) point (XYZ). Possible visual mismatches between their established views of the 2d and 3d space and the feedback received from the environment could serve as starting points for new conceptualizations.

#### *Phase 2: An arrangement of objects*

For the second phase of the experimentations, we asked the students to represent in the environment's 3d space an engineering component for which we only provided a 2d drawing (Figure 4a). This was supposed to be the drawing a client had given them, requesting a presentation of a 3d prototype, so as to give his approval for the component's final production in C.N.C machines. As the 2d drawing represented the Top View of the 3d component, the only dimensions defined in it were the ones visible from this specific orthographic view. The dimensions of the 3<sup>rd</sup> axis -and consequently the final shape of the component- were completely left to the students' choice. Since the cuboid they worked with in during the previous phase appeared to be now a main part of their component, we expected students to try to combine the cuboid's position in the 3d space with the positions of the rest stereometric objects necessary to complete the represented prototype. This process could foster the need to form specific spatial relationships between different 3d shapes, possibly performing at the same time rotations of the component in different directions so as to inspect it as a unified entity.

#### *Phase 3: Moving the tool on the XZ surface*

For the third phase of the experimentations we gave students a half-baked microworld (Kynigos,



2007) depicting, when running a Logo program, a machining procedure we presented as “planing across a cuboid’s four XZ edges”. However, after planning the first XZ edge, the tool was programmed to move in random ways inside the 3d space (Figure 5). Half-baked microworlds, being by their own nature incomplete, intrigue students to explore their functionalities deconstruct them and built on their parts. The “debugging” of a faulty machining Logo procedure could engage students in visualisation processes which may entail making sense of an object’s position and displacements (the tool’s) with regard to another one (the cuboid), both in terms of Cartesian geometry (3d coordinates) and Logo intrinsic geometry (distance from the previous position and heading).

### Data collection-Method of analysis

A screen-capture software (HyperCam2) was used to record the students’ interactions with the “3d Modelling & Cutting” computational environment, while a camera operated by the researcher was used to record the students’ gestures and the process of generating free hand sketches and 2d drawings on paper. The corpus of data also included the researcher’s field notes and the students’ answers on worksheets we provided at each phase of the experimentations. The video-recorded data from the screen-capture software were verbatim transcribed, while the rest of the data were used for providing additional details. In analysing the data, we searched for verbal exchanges between the students and interactions with the computational environment that indicated that students brought forth, utilized and enhanced their spatial and visualization abilities as they attempted to make sense of the 3d space and specify the absolute and relative positions of the objects represented inside it.

## Results

### Tracking an unfamiliar 3d space

At the first phase of the experimentations the students were given a 5x5x1 object placed in a random position inside the 3d space and were asked to turn it to 3x4x1 and then move it to X0 Y0 Z0. Although it was quite clear that this was a 3d object to be consequently shaped in a milling machine, the students chose to represent it in paper using a 2d (XY) frame of reference (Figure 3). The shape of this object corresponded to the static 2d orthographic view activated at that time on their computer screen (i.e. a rectangle 3x4). It was only after trying to manipulate the object inside the 3d space and move it to X0 Y0 Z0 that they looked for a 3<sup>rd</sup> axis (i.e. the Z) and defined a new, 3d frame of reference (XYZ), also determining each positive half-axis’s orientation in the 3d space.

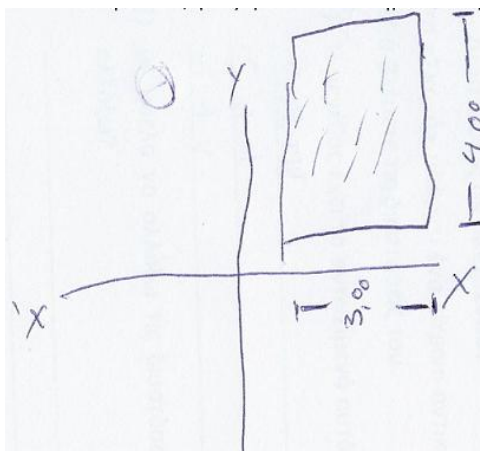


Figure 3. The 2d reference system the students originally came up with. The  $x'$  and  $y'$  are the negative half-axes

S2:  $Z = 0$  is here [points vertically to the XY surface on the paper]. Z equals to 0 at 1.

R: Would you like to explain that?

S2: Now we can't see it [the Z axis]. I guess that's because we look at it [the object] from above

S1: Z is... comes from above [points vertically to the screen]

S2 decides to move the cuboid to  $Z=0$ .

S3: What happened???

R: How did it move?

S1: Upwards!

S2: At the Y axis's direction! At its positive direction! .... I'll move it again. Let's make  $X=1$ ....[the object moves to the

*right]. That's normal for the X axis.*

*S1: I'd say this goes the other way around. That's the XZ there. Not the XY!*

*S2: Make Z -2!.... [after the feedback] That's rotated! [the reference system they had came up with]. Z defines upwards-downwards!*

Although recognising 3d frames of reference and working with 3d coordinates was part of their curriculum, mapping the “3d Modelling & Cutting” environment’s unknown 3d space seemed to challenge the students’ spatial orientation and visualization abilities. Initially, the students seemed to devise an egocentric X-Y frame of reference that corresponded to the way they were looking at the object on the screen from their own viewpoint (Figure 3). Disregarding the fact that this was a 3-dimensional object (3x4x1) and its position was defined by an ordered triple of coordinates (X, Y, Z), the frame of reference they formulated was merely a 2d one. However, being able to *dynamically manipulate* the object inside the 3d space and observe the effect of their actions seemed to be enough of a spark to trigger new visualization processes that incorporated the interpretation of the object’s changes of position in the 3d space. Translating the object’s displacements inside the 3d space and interconnecting them to the X, Y, Z axes, the students defined the environments’ frame of reference as being three dimensional and “rotated” with regard to the one they had originally come up with.

### Representing a 3d object using a 2d drawing

To complete the construction of their engineering component in the 3d space, we gave students a 2d drawing that represented the component’s Top View (Figure 4a). Although it was once again quite obvious that this was a 3d component that would be finally cut in CNC machines and that the “3d Modelling & Cutting” was a computational environment inside which 3d objects could be represented as such, the fact that we gave them a 2d drawing seemed to disorient and confuse the students.

*S3: Now, to represent the rest of the component's parts, these drawings [pointing at the circles at the 2d drawing]....*

*R: What are these?*

*S3: Circles*

*R: What they would be in the real component?*

*S3: Holes*

*S1: Holes*

*S3: And we want to represent them, right? What will we do?*

*S1: Use circles!*

With the Top View (XZ plane) being activated on the environment and looking at the 2d drawing, the students decided to represent the holes in the component not as cylinders (having some specific depth up to which they would have to move their drilling tool when machining them), but as 2d circular disks. The fact that the 2d drawing of the 3d component didn’t include any information about the 3<sup>rd</sup> axis’s dimensions, seems to have tricked students into transferring elements and properties of the 2d orthogonal projection into the 3d representation of the component (Parzysz, 1988). Even though this component was a quite common one in the engineering field (i.e. a flange) and the students were able to explain its function in specific machineries when asked, they kept representing with 2d disks all of its parts that the 2d orthogonal projection made them look circular. Finally, every hole or other cylindrical element on the component was represented merely as a 2d circular disk in the XZ plane (Figure 4b).

Taking about their client and the fact that he would be interested in the examining the component in detail before giving his approval for its production in CNC machines, the students looked for functionalities of the environment that would allow them to display the other two orthogonal views of the object, the Front and the Side one.

S1: [S1 moves the “Camera Controller” around and Figure 4b reveals]. What happened? The client won’t be satisfied! [laughing]... This shows us everything. All the views we need, the front, the top, the side.

R: What about the parts we inserted?

S1: We have to state that these are holes.

R: What kind of objects we’ll use for this?

S1: Solids, 3 dimensional. The circle is not one of them. Look how we got misled!!

R: What can we do?

S1: Let’s give the circle some height! [make the circle a cylinder].

Manipulating the Camera Controller, the students navigated in the 3d space and observed the shapes and positions of the component’s constituent parts, examining them from different viewpoints. The *dynamic visualization* of the component through the use of the Camera Controller enabled students realise that they had been “misled” by the static 2d drawing. It revealed the need not only to use 3d geometrical objects instead of 2d ones, but also to specify spatial relationships among the component’s parts that would not differentiate as they changed viewpoints inside the 3d space.

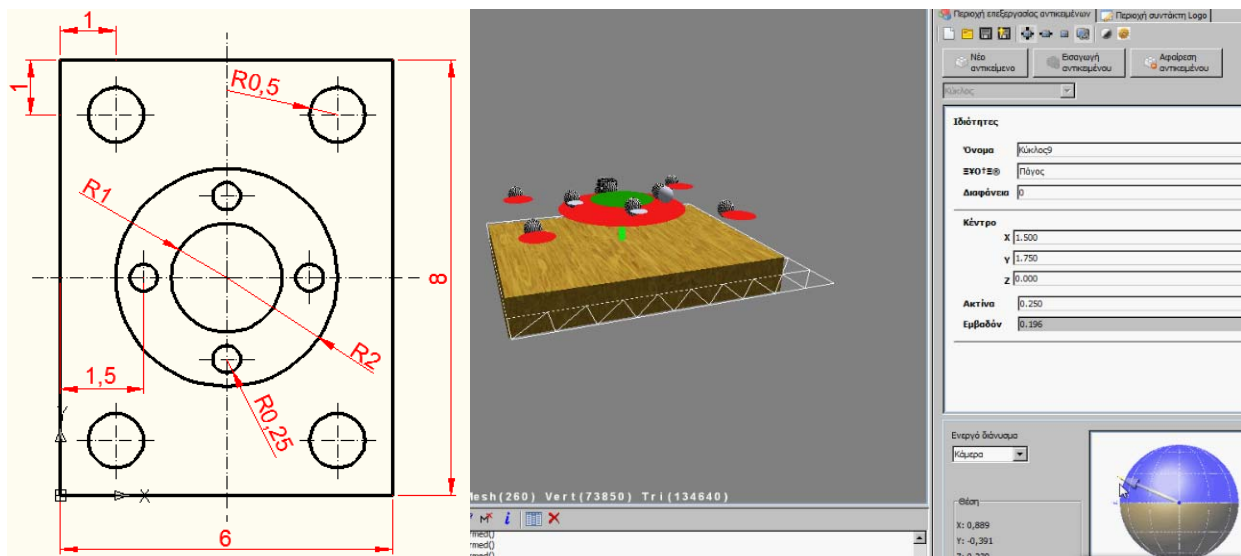


Figure 4. The 2d orthogonal projection of the component and the students’ initial 3d representation of it

### Using Cartesian and Turtle geometry to explain movements in 3d space

At the third phase of their experimentations, the students were introduced to a half-baked Logo microworld representing a “planing across the cuboid’s XZ edges” machining procedure. Planing is a common machining procedure during which a cutting tool removes material from a piece of metal, moving in linear paths, and is usually performed with the intention generate accurate and flat surfaces. However, in this microworld, as Figure 5 shows, the cutting tool, after executing the first edge’s planing, starts moving around in the 3d space in a random way, ending up plunged deep inside the component.



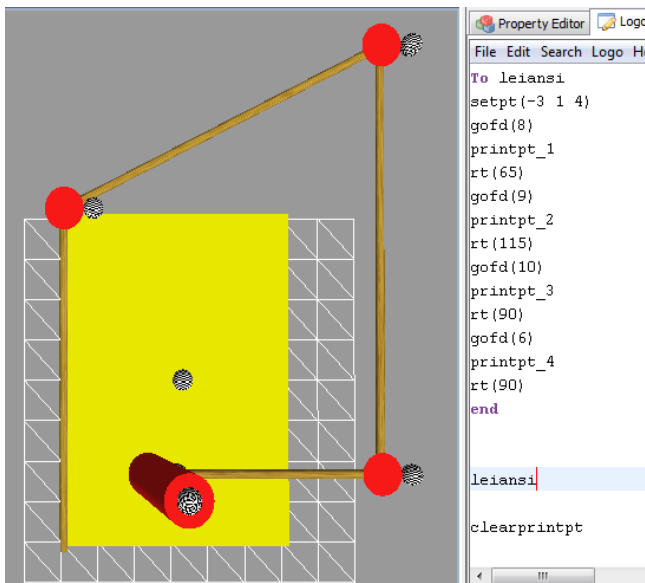


Figure 5. The graphical result of the half-baked “planing across the edges of the XZ plane”

S2: WHAT IS THAT? [Laughter]

R: It was supposed to plan all four edges.

S2: At least THERE ARE 4 edges! [more laughter]

S3: We’ll have to start from the beginning!

S2: We should CORRECT it right from the beginning!

As they attempt to correct the Logo procedure the students start talking about Cartesian geometry coordinates and Turtle geometry commands:

R: Where does the tool begin its movement?

S2: [reading the setpt command] X= -3, Z = 4. Here! [points at the lower left corner]

R: What happens next?

S2: Go forward 8.

S1: It goes to X -3 and Z... [takes a moment to add 8]...12....? It should be minus 4!!!

S2: It goes here [points at the upper left corner]. That’s X=-3 Y=0.5 Z=12. It’s another way to say that from the zero point [the previous one] it moved 8 in forward direction.

S3: Yes, but didn’t it move in the upwards direction? There is where the negative Zs are. Measure it from the centre. Minus 4. 8 is the whole piece.

Decomposing the Logo program to generate a correct planing procedure, the students use the “Turtle Geometry” commands so as to specify the tool’s Cartesian coordinates as it moves inside the 3d space. Logo environments have been considered to promote spatial thinking (Clements and Battista, 1992) as Turtle Geometry provides a different way of measuring and moving (in the 2d plane or 3d space), complementary to Coordinate Geometry (Abelson and diSessa, 1980). The students, working with a microworld where manipulating an object (the tool) and moving it inside the 3d space is not matter of just pressing a button, recognize a global frame of reference (“X= -3 Y = 0.5 and Z = 12”) and a local one (“8 in the forward direction from zero point”) that they use interchangeably to navigate inside the 3d space.

## Discussion

Mathematics educators for decades now develop and evolve dynamic computational environments that employ dynamic visualization of external representations as means for exploring and understanding mathematical concepts, both in Algebra and Geometry (CAS, DGEs, microworlds for constructionist learning). Visualization and spatial thinking in engineering education, however, are viewed as processes independent of the use of external representations and when computational environments are used, the black-boxes they contain slim down the opportunities for authentic 3d space explorations.

Computational environments, however, that incorporate the use of dynamic visualization and are developed under a constructionist perspective -along with purposefully designed collaborative activities- seem to provide a rich context for engineering students, allowing them to mobilize, enhance and develop elements of their spatial and visualization abilities. The dynamic manipulation of 3d objects inside these environments, as well as the dynamic manipulation of

the viewpoint from which the students may observe their constructions, may trigger visualization processes connected with the spatial orientation inside the 3d space and the formation of spatial relationships among constituent parts of 3d configurations of objects. Half-baked microworlds (Kynigos, 2007), may foster genuine explorations in the 3d space, as their main characteristic, the fact that they are incomplete by design, challenges students to decompose them and built on their parts, so as to make them work “correctly” or to produce a different artefact according to their own conceptualizations of the 3d space. Logo procedures provide a symbolic notational system that brings to light obscure mathematical procedures professional environments box up under UI buttons, degrading visualization processes that could be fruitful for engineering students. It goes without saying that further research on constructionist dynamic computational environments designed to support engineering students’ visualizations in the 3d space is needed to enrich these findings.

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