Constructionism applied in early childhood mathematics education: Young children constructing shapes and meaning with sticks.

Chrystalla Papademetri, *C.Papademetri@euc.ac.cy*
Dept of Education Sciences, European University Cyprus

Abstract

Constructionism has been traditionally connected with computer-based research. This is probably why there is limited research within constructionism involving young children. Thus, we consider that the contribution of the study described in this paper relates mostly to the age of the subjects in conjunction with the tools employed. The aim of the study is to describe and analyze young children's understandings of shapes through an investigation of squares. A more focused purpose of the study is to investigate what knowledge young children have about the structure of simple shapes, how this knowledge is expressed, and how it is used in the process of constructing squares. The youngest child out of the 52 involved in the study was four years and ten months old and the oldest was six years and eight months old and for the construction process involved the tools employed were wooden sticks.

The consensus in existing literature is that children's limited, and often appearance-based descriptions of shapes, indicate that children view shapes as a whole and lack structural understanding. This study approaches children's understandings of shapes from a different perspective, based on an alternative and more dynamic interpretation of the van Hiele model and with the acknowledgement that there might be multiple ways of knowing and expressing mathematical knowledge. This study examines the understandings young children have about the structure of shapes. The methodology designed for the study is based on the constructionism idea of learning-by-making which leads to the need to search for tools that will act as objects-to-think-with and allow learners to communicate their thinking-in-change.

Fifty-two, five to six year olds, were engaged in three phase naturalistic task-based interviews. In Phase A (Description) the children were involved in classification and shape recognition activities. In Phase B (Construction) the children were asked to construct squares with the use of sticks and, in Phase C (Reflection) the children were asked to reflect on the construction process of Phase B. Even though during Phase A, the children, as supported by existing research, exhibited limited structural understanding about squares, through their involvement in Phase B, they exhibited much richer intuitive structural understandings. In Phase C, children tended to express structural understandings about squares in diverse and inventive ways.

The findings challenge the view that children's limited verbal descriptions of shapes indicate lack of structural understanding. In the process of the interviews the children articulated, through the 'language' provided, structural knowledge about squares that may be characterized as intuitive - if we share DiSessa's, definition of intuition (DiSessa 2000)- and at the same time they were able to situate their abstractions in the context of construction. Overall the findings indicate that, provided sufficiently sensitive techniques are employed, it is possible for children to express structural knowledge in diverse and often unconventional ways.

Keywords

Young children, shapes, construction, intuitive knowledge, non-verbal thinking

Introduction

Constructionism builds on the simple but powerful idea that with the appropriate tools learners can 'build things and ideas simultaneously' (Noss & Hoyles, 2006) and 'evokes the idea of learning-by-making' (Papert, 1991). This powerful idea that was added to the constructivist connotation that knowledge is actively constructed by the learner, has provided access to the construction of new meanings. Since, as supported by Noss and Hoyles (1996), it was the computer that allowed 'glimpses to new epistemologies' and 'opened new windows on the construction of meanings' it is not surprising that constructionism has traditionally been connected with computer-based research. This is probably why there is limited research within constructionism involving young children. The involvement in computer-based tasks requires a certain familiarity with the mean that might be considered as an agent which adds extra exertion that discourages researchers from investigating constructionism with younger children.

Thus, we consider that the contribution of the study described in this paper relates mostly to the age of the subjects in conjunction with the tools employed. The aim of the study is to describe and analyze young children's understandings of shapes through an investigation of squares. A more focused purpose of the study is to investigate what knowledge young children have about the structure of simple shapes, how this knowledge is expressed, and how it is used in the process of constructing squares. The youngest child out of the 52 involved in the study was four years and ten months old and the oldest was six years and eight months old and for the construction process involved the tools employed were wooden sticks.

Review of the literature

Young children's understandings of shapes

The consensus in existing literature (Burger, 1985; Burger & Shaughnessy, 1986; Shaughnessy & Burger, 1985; Clements et al, 1999; Clements & Sarama, 2000; Clements et al, 2001; Clements & Battista, 1992) which has formed the picture of what is mostly believed about young children's understandings of shapes is that children's limited and often appearance-based descriptions of shapes indicate that children view shapes as a whole and lack understanding of shape structure. It is interesting to search for the origins of this consensus. The attempt to discuss the origin of research concerning geometric thinking leads to the van Hieles.

Many years ago, when I found myself reading 'Structure and Insight' (van Hiele, 1986), at first I only managed to get to page 9 were van Hiele strongly claims that 'thinking without words is not thinking'. I rebelled; the van Hiele model was founded on a claim I could never accept. Because of my everyday experiences with young children I could never agree with van Hiele's claim. My experiences led me to the empirical realization that children are capable of, and understand so much more, than what they can express in words. My motive for getting involved in research on young children and shapes was exactly this realization which arose from the many opportunities I had of observing preschoolers constructing shapes when they 'had no knowledge' (if we measure knowledge by verbal language) of concepts like right angle, parallel lines etc. A realization which is highly supported by the consensus within early childhood education research that, 'the child has a hundred languages and a hundred hundred more. But they steal ninety-nine the school and the culture. To think without hands to do without head … (Loris Malaguzzi, poem translated by Lella Gardini, in Edwards et al, 1998). This poem came into my mind when I came across van Hiele's claim that 'thinking without words is not thinking'. The research culture which led to the consensus that children's limited and often appearance-based descriptions of shapes indicate lack of structural understanding, certainly fits the culture described by the poet and is highly connected implicitly or explicitly with van Hiele's claim that 'thinking without words is not thinking'. Within this research culture children were 'assessed' based on what they said.

According to van Hiele (1986) himself, at the first level of his proposed model of geometric thinking, children will simply say: ‘This is a square’, without any further explanation and without ‘being able to mention even one property of’ the shape (p.62). Van Hiele-based research has interpreted Level 0 as the level where children simply ‘visually’ recognise and describe shapes based on their appearance (‘This is a square because it looks like a window’), see shapes as a whole, pay no attention to and have no understanding of shape properties. This interpretation of van Hiele’s level 0 is widely accepted even though, if we carefully ‘read’ van Hiele’s description of the level, he is not saying that children’s inability to express in words excludes structural understanding. He is simply describing the nature of children’s utterances. Similarly my experience in relation to children’s attempts to construct shapes led me to the hypothesis that judging a shape by its appearance does not exclude structural understanding.

Even though my first reaction when I got to page 9 of ‘Structure and Insight’, was to close the book, I soon realised that I could not claim to be studying children’s understandings of shapes without reading the whole thing. Almost every existing study on geometry refers to the van Hiele model of geometric thinking. I could, therefore, not ignore it and base my study solely on other researchers’ interpretations. So I returned to the book and gradually realised that the theory unfolded by van Hiele had nothing to do with his opening claim that ‘thinking without words is not thinking’. For instance van Hiele glorifies the importance of intuition in the process of thinking as he devotes a substantial part of ‘Structure and Insight’ to intuition which he defines as a way of ‘seeing the solution to the problem directly, but without being able to tell’ (p. 76). It is noteworthy to mention that van Hiele himself in a paper published in 1999 admitted that ……

> …… thinking without words is not thinking. In Structure and Insight (van Hiele, 1986), I expressed this point of view, and psychologists in the United States were not happy with it. They were right. If nonverbal thinking does not belong to real thinking, then even if we are awake, we do not think most of the time. Nonverbal thinking is of special importance; all rational thinking has its roots in nonverbal thinking, and many decisions are made with only that kind of thought. In my levels of geometric thinking the ‘lowest’ is the visual level, which begins with nonverbal thinking (p.311).

The fact is that van Hiele’s positive stance towards non-verbal thinking was obvious in ‘Structure and Insight’ in spite of his negative claim regarding non-verbal thinking. ‘Structure and Insight’ was highly characterized by paradoxes which van Hiele-based research failed to detect. A thorough discussion of these paradoxes and an analysis on how they have influenced van Hiele-based research is provided in Papademetri (2007). Additionally, significant dynamic aspects of the van Hiele theory which were eliminated by van Hiele-based research are emphasized. This attempt to re-visit van Hiele in Papademetri (2007) led to the conclusion that van Hiele-based research failed to ‘read’ van Hiele, and as a consequence substantial pieces of research concerning geometric thinking are characterized by a tendency (a) to evaluate children mainly through their verbal ability, (b) to emphasize children’s ‘wrong responses’, ‘misconceptions’; this led to a downplaying of children’s important, rich, intuitive understandings and (c) to evaluate the cognitive act with no reference to important aspects of the setting (e.g. child-adult interaction, activity) in which the cognitive act takes place; as an extension to this remark, young children were assessed in settings which were designed without taking into consideration the particularities of their age. A particular implication of these orientations is that they led to a restricted view of what children know about shapes. This restricted view has tended to degrade children’s structural understandings.

In the next part of this paper I will describe how constructionism can provide alternative routes towards a more ‘equitable’ investigation of young children’s understandings of shapes.

Construction, young children and shapes

As a starting point, this study values Hoyles (2001) insistence, on the conviction 'that studies in mathematics education should involve some discussion of mathematical activity' and that the knowledge constructed in children's heads is highly connected with the tools at hand. This conviction has its roots in the Vygotskian assumption that 'the activity in which knowledge is developed is an integral part of what is learned' and on the belief that 'treating knowledge as an integral, self-sufficient substance, theoretically independent of the situations in which it is learned and used' assumes 'a separation between knowing and doing', (Brown et al, 1989).

Noss & Hoyles (1996), within the framework of computer-based research emphasise the 'need to focus on tools and settings' as well as 'on the ways in which the understanding of mathematical ideas is mediated by the tools available for its expression' (p.50). Thus we have the constructionism principle that learning takes place in situations where learners are allowed to build and reflect on their own models (Kafai, 2006) which builds on the powerful idea of thinking-as-constructing which suggests that actual, physical construction can lead children to new understandings. 'Constructionism suggests that learners are particularly likely to make new ideas when they are actively engaged in making some kind of external artifact – be it a robot, a poem, a sand castle, or a computer programme – which they can reflect upon and share with others (Kafai & Resnick, 1996, p.1). Similarly, Noss & Hoyles (1996) refer to the importance of focusing on 'the ways in which the understanding of mathematical ideas is mediated by the tools available for its expression'. This leads to a search for 'objects-to-think-with', to borrow Papert's (1993) expression, but also tools that can be used by children as a language to express and communicate their 'thinking-in-change', to borrow an expression by Hoyles (2001). This study builds on the perspective that communicating is an integral part of thinking. Thus, in designing activities, there is a need for providing children with what Noss & Hoyles (1996) define as an 'autoexpressive' language; a language which acts both as a thinking tool and an expressive tool.

Even though Vygotsky (1962) states that 'to understand another's speech it is not sufficient to understand his words …we must understand his thought' (p.51), as we saw in the previous subchapter of this paper there is a tendency of restricting language to speech, to a verbal communication system. Within the computer age and Papert's revolutionary import of the programming language and the idea of 'thinking in images' (Papert, 1986), the discussion about language and thinking is elevated to a different level and context. Until before 'new technologies' opened new roads to thinking and understanding, children's use of words was analysed in order to unravel what children know or do not know, what children can or cannot do. But what the computer revolution showed was that language (the process of communicating and expressing) is not only about words. Words are not the only carriers of meaning and knowledge.

The study described in the remaining of this paper aims to add to this attempt to investigate the hypothesis that children might think in alternative ways, and challenge the idea that thinking depends on language (thinking in words). In addition, whereas existing studies which seem to have played an important role in formulating a picture of young children's understandings of shapes were build on an assumption of 'what thinking without words is not' the study described in this paper aspires to investigate 'what thinking without words is'. In addition it aspires to allow young children to 'look through' the same windows on the construction of mathematical meaning which were opened by constructionism. Thus, the effort was to design a study that would allow us to overcome the restrictions within most existing research as these were described in the previous section of this paper, in an attempt to better describe and analyse young children's understandings of shapes. To be more precise, the aspiration of the study was to investigate what knowledge young children have about the structure of simple shapes, how this knowledge is expressed and how it is used in the process of constructing shapes. The emphasis thus, was placed on the use of construction as a methodological tool.

Methodlology

In order to address the aim of the study, 52 children were engaged in task-based interviews, consisting of a three-phase framework (Description-Construction-Reflection). In Phase A-Description Task (DT), the children were involved in classification and shape recognition activities. This opening phase enabled subsequent data to be evaluated in comparison to those of existing research. In Phase B-Construction Task (CT), the children were given wooden sticks of various lengths and were asked to construct squares. This allowed the children to express their understandings of shapes in alternative ways. Finally, in Phase C-Reflection Task (RT), the children were involved in a process of reflecting on the construction process of Phase B.

Special attention was paid in relation to designing a research appropriate for the age of the subjects. During the research design, I had to constantly keep in mind the fact that this study was both a study within the domain of mathematics education as well as within that of early childhood education. As stressed out in literature on early childhood education research (Brooker, 2001; Donaldson, 1978; Grieve and Hughes, 1990; Tizard and Hughes, 1984; Wells, 1985) it is 'commonsense' knowledge (for people familiar with the nature of young children) that it is more likely to penetrate into children's minds if you investigate them in a familiar environment, with familiar adults. That is why it is a striking experience for people with this kind of familiarity with young children to come face to face with research concerning young children, when such research ignores commonsense knowledge of how young children think and act. Therefore, in an effort to enable children to communicate in an authentic way, special arrangements were made that allowed naturalistic elements into the setting of the interviews.

Thus the interviews were conducted by a group of thirteen student teachers as part of a teacher training course they were attending at the University of Cyprus. The interviews were conducted in two public schools where the student teachers were 'working' as pre-service teachers. The student teachers attended a training program and were provided with a detailed interview scrip tool. The training program and the interview script tool were the products of an iterative piloting procedure completed in three cycles. The involvement of the student teachers allowed the inclusion of naturalistic elements in the research design and enabled the children interviewed to express their understandings while being involved in activities with familiar adults and in familiar settings. It also allowed for a far greater sample than would have been possible otherwise. All interviews were conducted in the children's own school as part of their everyday involvement in free play activities where children are freely engaged in controlled activities. Given the many variations of play to which the children involved in the study were already accustomed within free play settings, the task-based interviews were considered by them as yet another usual activity.

All interviews were videotaped and transcribed. The coding scheme that was used for data analysis was developed in two stages. A preliminary coding scheme was developed with the use of the data collected from the piloting procedure. This was based on an initial open coding process in an effort to identify interesting phenomena and patterns among the data. This was then revised and advanced with the use of the data collected from the main study.

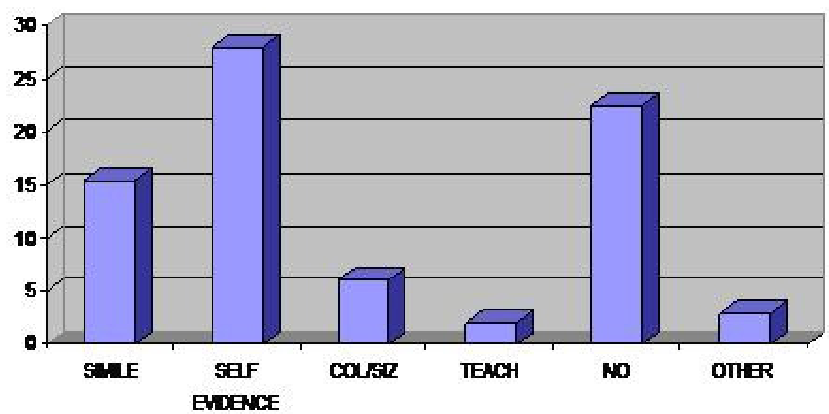
The findings of the study as presented in the following section of this paper aim to provide an insight on how the study's participants used construction (a) to communicate rich, intuitive, structural understanding about squares and (b) as a tool-to think-with. In order to address these two issues a brief reference will be made to the findings of the DT and a more extensive reference will be made to the findings of the CT. In the CT the children had up to three attempts in order to construct a square. In this paper special attention will be given to the children's first attempt to construct a square. Additionally we will refer to some of the children's second attempt.

Findings

Children expressing their understandings through Description

Before presenting the findings in relation to how the children involved in the study used construction in order to communicate their understandings about squares, it is important to briefly describe the findings in relation to the children's involvement in the DT. One hundred and thirty-eight responses were collected from the fifty-two children that took part in the study. The information provided here provides a first sense of the 'quality' of the children's understandings as expressed in a setting restricted to classification, recognition and description tasks.

During the data analysis process it was considered essential to define a system which would allow the distinction between expression means which implicitly included structural elements ('reference' to the shape's structure) and other means of expression. It is interesting to note that within the setting of the DT, 77% of the children's responses did not implicitly include any structural elements. Graph 1 presents the findings in relation to the categories of responses which were categorized as non-structural answers. More than 25% of the non-structural responses included a self-evident justification ('because this is how a square is', 'because they have a square shape'). Besides the responses which included a self-evident justification, there was a significant number of responses which were categorised as 'NO' responses. In these cases, children would state that they didn't know anything about squares or they would simply not reply to the interviewer's question. There were 31 such responses all together (Graph 1, NO). Additionally, 16% of the children's responses in this study included a simile (Graph 1, SIM). The children would say that squares are like 'a house', 'a carpet', 'windows', 'the underneath of a house'.



All values are given in percentage rates.

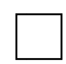
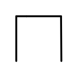
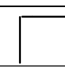
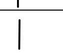
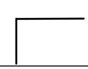
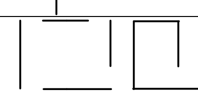
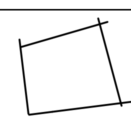
Graph 1: Results in relation to the sub-categories of responses which were not classified as non-structural

If this study was to follow the same methodological framework as existing studies on young children and shapes it could reconfirm the existence of van Hiele level 0. Shaughnessy & Burger (1985) claim that when younger students where asked what they would tell a friend so that s/he could pick up all squares from a sheet of paper they would answer 'I'd tell them to pick out all the squares' or 'look for the doors'. Consequently, Shaughnessy & Burger (1985) reconfirmed the existence of van Hiele level 0 as the level where descriptions are purely visual and no attention is given to shape properties. The difference between this study's methodology and the methodology followed by Shaughnessy & Burger (1985) lies in the fact that here the aim is to describe the ways children express their understandings of shapes within and in correlation to a specific setting and not to evaluate children in order to place them in levels independently of the setting in which they express their understandings. In conclusion, one can argue that within a setting restricted to simple classification, recognition and description tasks, children exhibited poor structural understandings of shapes. We will now describe children's understandings of shapes as these were expressed through their attempt to construct a square.

Children expressing their understandings through Construction

In my effort to identify an analysis tool for analysing the data collected from the CT, I faced a practical problem. There were some commonalities in relation to some actions in the children's attempt to construct a square, but the route each child followed was unique. The case of two or more children following exactly the same process was rare and thus, categorizing fifty-two children into 20-25 categories was not considered an effective way to categorise the data. So what could be the criterion for a meaningful categorisation? To answer this question, a very careful and repetitive examination of the data collected was considered essential. What became apparent from carefully studying the raw and transcribed data was that children would base their construction on a specific choice, **a foundational action** that had the attribute of stability. This specific action, which involved the choice of specific sticks and/or their spatial arrangement, was an action that remained intact until the end of their attempt and exhibited understanding of a specific property (or properties) of a square. This foundational action was the children's first action in their attempt to construct a square. The children would then proceed with other choices and actions in order to complete their construction. All of these other choices and actions during the construction attempt involved the element of experimentation and thus indicated that children were in a constructing process of building new knowledge on their original intuition. The identification of the children's foundational actions in the CT allowed the identification of nine square construction strategies among the fifty-two children that participated in the study. These nine categories are described in Table 1.

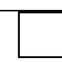
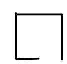
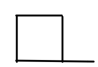
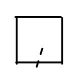
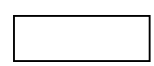
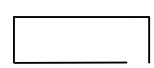
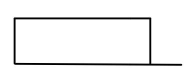
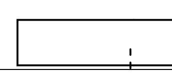
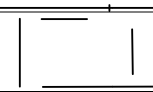
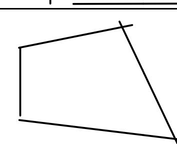
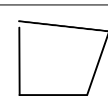
Table 1 Description of the strategies the children followed during their attempt to construct a square

Strategies	**Code**	**Foundational Action**		
		Verbal Description. The child ……	**Choice of Sticks**	**Spatial Arrangement**
1	**S1**	…… selects four equal sticks and places them one by one creating right angles and thus constructing a square.	4 equal sticks	
2	**S2**	…… selects three equal sticks and constructs an open shape with right angles.	3 equal sticks	
3	**S3**	…… selects two equal sticks and constructs a right angle.	2 equal sticks	
4	**S4**	…… selects two equal sticks and places them parallel and aligned.	2 equal sticks	
5	**S5**	…… randomly selects two (unequal) sticks and creates a right angle.	2 unequal sticks	
6	**S6**	…… randomly selects four (unequal) sticks and tries to construct a four-sided shape with four right angles.	4 unequal sticks	
7	**S7**	…… selects three equal sticks.	3 equal sticks	**-**
8	**S8**	…… selects two equal sticks.	2 equal sticks	-
9	**S9**	…… selects one stick at a time and tries to construct an irregular quadrilateral which somehow looks like a square with its sides not equal and its angles not right.	4 unequal sticks	

Besides the patterns identified within the data which allowed the identification of the nine stategies described in Table 1, patterns and commonalities were also identified in relation to the final product of the children's first attempt to construct a square. Eleven categories were identified in relation to this aspect. The products of the children's attempt to construct a square are described in Table 2 and are classified into three types (Table 2, Type A, B, C).

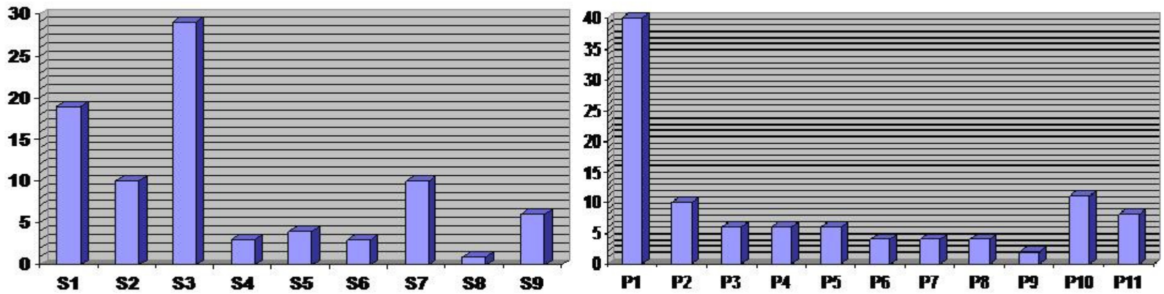
First, let us take a look at which Strategies (S) the children followed and what Products (P) they ended up with in their first attempt to construct a square. Graph 2 shows the percentage of children that followed each of the S identified and described in Table 1 and the percentage of children which ended up with each of the P as these were described previously in Table 2. A first significant observation that can be made based on Graph 2 is that the percentage of children that followed S1, S2 and S3 is much higher than that of the children that followed other strategies. As we can see in Graph 2, the most frequently used strategy among the 52 children was S3. 29% of the subjects used this strategy, in other words began their construction by selecting two equal sticks and constructing a right angle. A significant percentage of children (19%) followed S1 (selected four equal sticks and constructed a square with no experimentation required). The rest of the strategies were followed by smaller groups of children.

Table 2 Description of the products of the children's attempt to construct a square

Type	Product	Code	Verbal Description	Graphical Description
A	1	**P1**	Square with four equal sticks.	
	2	**P2**	Square with four sticks (gap).	
	3	**P3**	Square with four sticks (extension).	
	4	**P4**	Square with more than four sticks.	
B	5	**P5**	Rectangle with two sets of equal sticks.	
	6	**P6**	Rectangle with four sticks (gap(s)).	
	7	**P7**	Rectangle with four sticks (extension).	
	8	**P8**	Rectangle with more than four sticks.	
	9	**P9**	Rectangle with four sticks (gaps and extensions).	
C	10	**P10**	Irregular quadrilateral that resembles a square but has no right angles.	
	11	**P11**	Irregular quadrilateral with some angles right and/or some sides equal.	

The slightly oblique dotted line crossing through the side of a construction indicates that that specific side is constructed with the use of two sticks.

As far as the products of the children's attempt to construct a square, one first key observation is that there is a big difference between the findings in relation to P1 compared to other products. It is quite astonishing that 40% of the children involved in the study (21/52) successfully constructed a square by using four equal sticks. The rest of the products were the result of the effort of much smaller groups of children.



Graph 2 Results in relation to the Strategies (S) followed by the children and the Products (P) of their attempt to construct a square

In Table 2, the eleven products identified among the data were categorised into three groups: Type A, B and C. What is important to highlight in relation to Graph 2 is that the majority of the children (62%) constructed a Type A shape (shape with four equal sticks/gap/extension/more than four sticks). Thus, whereas in the DT only 35% (18/52) of the children gave structural responses, 62% of the children involved in the study (32/52) constructed a Type A product in their first attempt to construct a square.

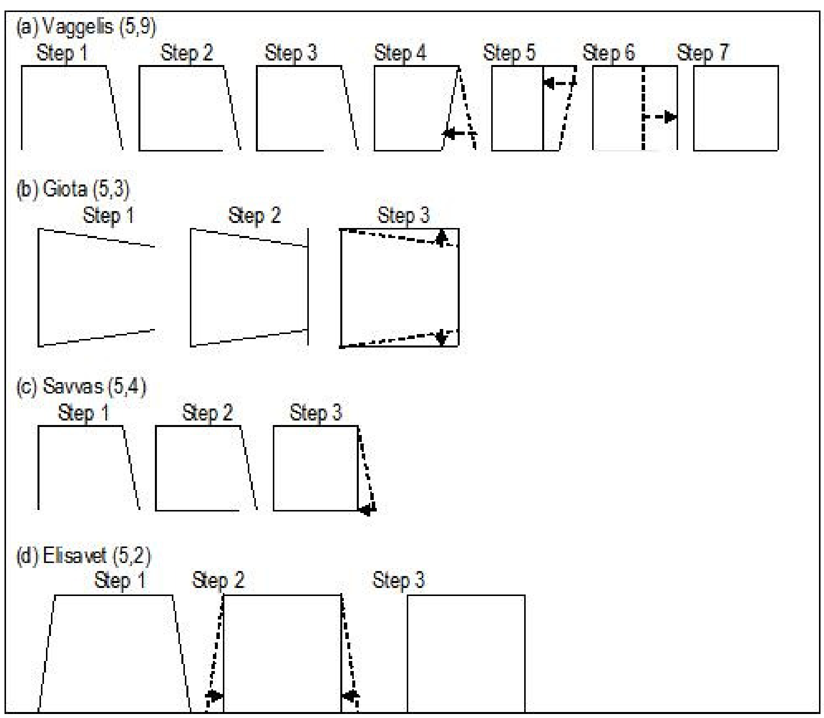


Figure 1 The construction routes of children who followed S7

During the process of data analysis (Chapter 5), two important observations were documented. First of all, that children following the same strategy did not necessarily end up with the same product and second, children that did follow the same strategy and end up with the same product did not necessarily follow the same route along the way. These observations reflect the variability that existed among the data. This variability is evident in the example routes in Figure 1. In Figure 1, we have an illustration of the construction routes followed by children which began their attempt to construct a square with S7. Even though all of these children followed the same strategy and ended up with the same product, followed different routes.

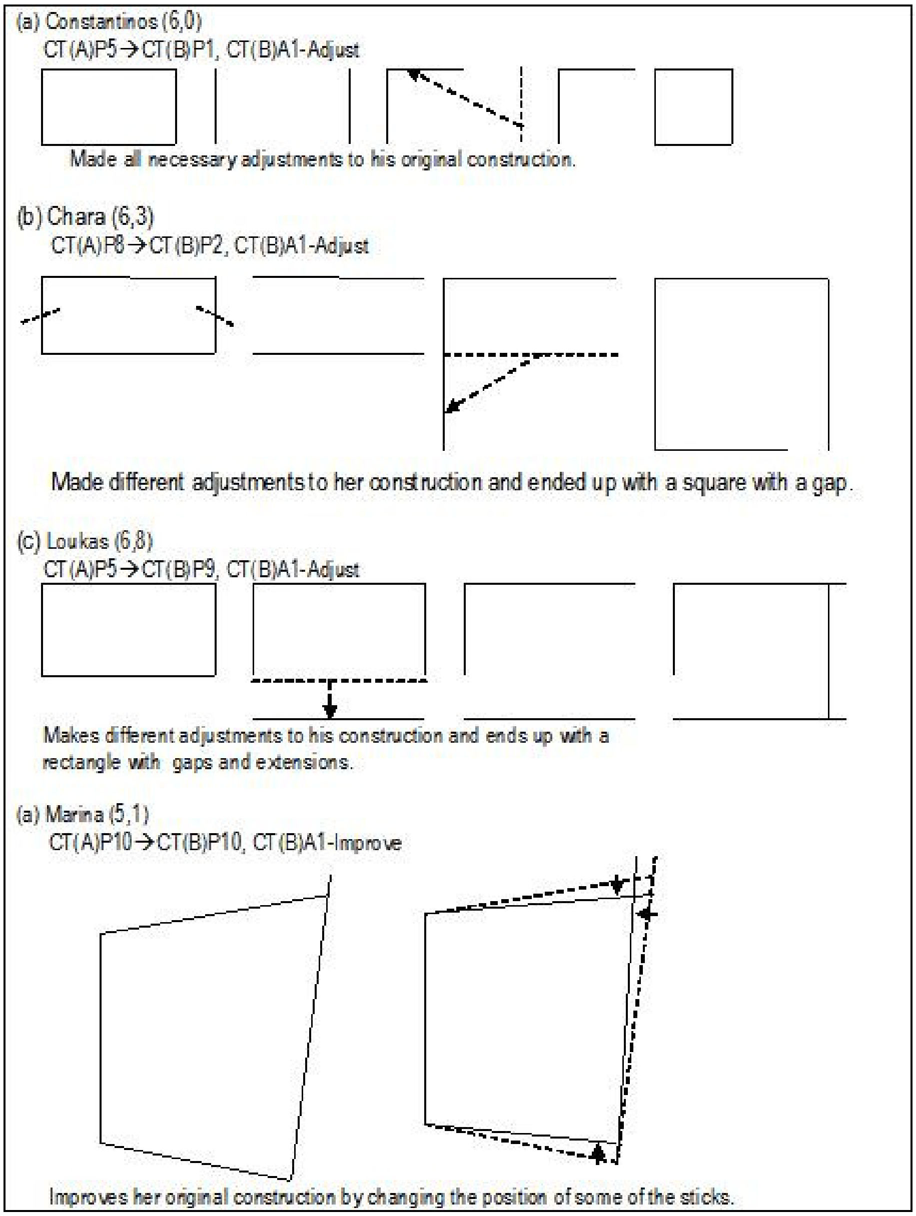


Figure 2 Some examples of the routes the children followed in their second attempt to construct a square

In the remaining of this section of the paper we will describe the routes followed by some of the children in their second attempt to construct a square (Figure 2) in an attempt to address the issue of how the children used the construction as an object-to-think-with and a tool for communicating their thinking-in-change. We had the cases of children like Loukas, for example. Loukas had constructed a rectangle in his first attempt to construct a square. After being encouraged by the interviewer to try again, he followed the route illustrated in Figure 2c. He ended up with a shape that looked more like a square than his original construction in the sense that the distance between the two vertical parallel sticks was more similar to the distance between the two horizontal parallel sticks. But the distance was still not equal. In many cases the children's actions allowed me to think of squares in ways I have never thought and look at square properties from a different perspective. Through his actions, Loukas gave a new perspective in relation to the square properties. Another way of expressing the equality of the sides is that the distance between the two sets of parallel sides has to be equal.

So Loukas acquired a vague intuition in relation to the distance between the sets of parallel lines contrary to other children such as Costantinos and Chara (Figure 2a,2b) who acquired a more clear intuition of the fact that it is not only the parallel sides that have to be equal but the adjacent sides as well. Similar to Loukas' was the case of Marina (Figure 2d). In an effort to make her original construction (quadrilateral with no right angles) look more like a square, Marina tried to close the angles. But the angles of her second attempt were still not right angles.

Construction versus Description

At first sight of the findings, one can argue that the children based their attempt to construct a square on specific structural understandings. These structural understandings unfolded through the children's strategies and products. Through the course of the children's attempts one can sketch the ways in which the children's understandings evolved and changed. The picture sketched in relation to the children's understandings as these were expressed through construction is rather different to the equivalent picture sketched from the children's involvement in the DT. Thus, at this point, we can support the point of view that even though within a setting restricted to description the children exhibited poor structural understandings about squares, through their involvement in the CT they exhibited rich structural understandings.

Discussion

As I claimed at the beginning, the aim of this study was to add to an attempt to investigate the hypothesis that children might think in alternative ways, and challenge the idea that thinking depends on words. In addition, the study described in this paper aspires to investigate 'what thinking without words is'. This last remark is an issue I would like to address in this last part of the paper in light of the findings as these were described in the previous section.

In order to address the question of what children know about squares first we need to address another important question. What should be used as an indicator of what children know: the strategy they followed, the whole route or the product of their attempt? One cannot ignore the fact that there was no linear connection between a strategy and a product. For example: the children that exhibited an understanding of the fact that a square has four equal sides and four right angles through the strategy they followed did not necessarily end up with a ('perfect') square at the end of their attempt. Should the children's 'failure' to construct a square at the end of their attempt erase the structural understandings they exhibited through the strategy they followed at the beginning of their attempt? Like in the case of Christoforos (Figure 3). Should his failure to construct a shape with for equal sticks erase the fact that he used three equal sticks to begin his construction? In addition, some of the children that exhibited an understanding of the fact that a square has four equal sides and four right angles at the beginning of their attempt and ended up with a square at the end of this attempt had to experiment. Again, does this experimentation imply that we should ignore the understandings these children exhibited at first?

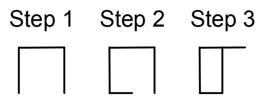


Figure 3 Christoforos (5,4) first attempt to construct a square

So here we are, as it shows, faced with a dilemma. Where should we focus on in order to determine children's existing knowledge: On their strategies, which included some 'correct' choices, on their need to experiment or on their sometimes 'faulty' products? Is this really a dilemma that needs to be resolved or is it a finding in itself? During the attempt to construct a square the children did not think in conventional ways. For example to think that '*a square has four equal sides and four right angles, thus I will select four equal sticks, place them in such a way as to construct right angles and thus construct a square'* in words, is a way of thinking in the conventional sense. It is a widely acceptable and recognisable way of thinking. If there was evidence among this study's data that children did think in such ways no one could deny that these children were thinking and it would be easy to identify exactly what they knew. But it is clear from the data that, in most cases, the children did not think in such ways. It is also clear that the children 'knew' specific aspects of a square's structure.

The question is what the nature of this knowledge is. The knowledge the subjects exhibited reminds us a lot of the way diSessa (1988, 2000) defines intuition. According to diSessa intuition constitutes 'little' pieces of knowledge, lack of systematicity and commitment, is very difficult to express into words, is 'rich', 'flexible' and 'diverse', and thus 'generative', is unstable, frequently effective and sometimes even correct. In light of diSessa's definition, it is quite safe to describe Christoforos understandings (Figure 3) as intuitive. In light of the study's findings we can support the point of view that through their involvement in the CT, the children exhibited a rich intuitive structural understanding of squares, and define intuition as the fragmented knowledge which children bring with them in a learning situation and are mostly able of expressing in ways which are contrary to what is formally acknowledged as correct.

Overall the findings indicate that, provided sufficiently sensitive techniques and 'languages' are employed, it is possible for children to express rich, intuitive structural knowledge in diverse ways. The search for such languages has been a main focus of computer-based research. But in this study, such a language was identified without the use of computers. Construction, the simple use of wooden sticks, became the language which the children could 'speak' and the adults could 'hear'.

References

Brooker, L. (2001). *Interviewing Children.* In G. Mac Naughton, S. A. Rolfe & I. Siraj-Blatchford (Eds.), Doing Early Childhood Research: International Perspectives on Theory and Practice (162-177). Buckingham: Open University Press.

Brown, J. S., Collins, A. & Duguid, P. (1989). *Situated Cognition and the Culture of Learning.* Educational Researcher, 18 (1), 32-42.

Burger, F. W. (1985). *Geometry.* Arithmetic Teacher, February, pp. 52-56.

Burger, W. F. & Shaughnessy, M. J. (1986). *Characterizing the van Hiele Levels of Development in Geometry.* Journal of Research in Mathematics Education, 17(1), 31-48.

Clements, D. H. & Battista, M. T. (1992). *Geometry and Spatial Reasoning.* In A.D. Grouws (Ed.), Handbook of Research on Mathematics Teaching and Learning (pp 420-464). New York: MacMillan Publishing Company.

Clements, D. H. & Sarama, J. (2000). Young *Children's Ideas about Geometric Shapes.* Teaching Children Mathematics, April 2000, 482-488.

Clements, D. H. Sarama, J. & Wilson, D. C. (2001). *Composition of Geometric Figures.* Proceedings of the 25th Conference of the International Group for the Psychology of Mathematics Education, Utrecht, 2, 273-280.

Clements, D. H., Swaminathan, S., Zeithler-Hannibal, M. & Sarama, J. (1999). *Young Children's Concept of Shape.* Journal of Research in Mathematics Education, 30(2), 192-212.

DiSessa, A. (1988). *Knowledge in Pieces.* In G. Forman & P. Pufall (Eds.), Constructivism in the Computer Age (49-70). Hillsdale, New Jersey: Lawrence Erlbaum Associates.

DiSessa, A. (2000). *Changing Minds.* USA: The MIT Press.

Donaldson, M. (1978). *Children's Minds.* Great Britain: Fontana Press.

Edwards, C. P., Gardini, L. & Forman, G. E. (1998). *The Hundred Languages of Children: The Reggio Emilia Approach.* Greenwich: Ablex Publishing Corporation.

Grieve, R & Hughes M. (1990). *An Introduction to Understanding Children.* In R. Grieve & M. Hughes (Eds.), Understanding Children (1-10). Oxford: Blackwell.

Hoyles, C. (2001). *From Describing to Designing Mathematical Activity: The Next Step in Developing Social Approach to Research in Mathematics Education?* Educational Studies in Mathematics, 46 (1-3), 273-286.

Kafai, Y. B. (2006). *Constructionism.* In K.R. Sawyer (Ed). Cambridge Handbook of the Learning Sciences. Cambridge:CUP.

Kafai, Y. B. & Resnick, M. (1996). *Introduction.* In Y. B. Kafai & M. Resnick (Eds.), Constructionism in Practice: Designing, Thinking and Learning in a Digital World (1-8). New Jersey: Lawrence Erlbaum Associates.

Noss, R. & Hoyles, C. (1996). *Windows on Mathematical Meanings.* Netherlands: Kluwer Academic Publishers.

Noss, R. & Hoyles, C. (2006). *Exploring Mathematics through Construction and Collaboration.* In K.R. Sawyer (Ed). Cambridge Handbook of the Learning Sciences. Cambridge:CUP.

Papademetri, C. (2007). *Structural Knowledge about Shapes: A Case Study of Young Children Describing, Constructing and Reflecting on Squares*. PhD Thesis, University of London, Institute of Education, London, U.K.

Papert, S. (1986). *Beyond the Cognitive: The other face of Mathematics.* Proceedings of the 10th Conference of the International Group for the Psychology of Mathematics Education, London, 1, 51-58.

Papert, S. (1991). *Situating Constructionism.* In I. Harel & S. Papert (Eds.), Constructionism (1-12). Norwood: Ablex Publishing Corporation.

Papert, S. (1993). *Mindstorms: Children, Mathematics and Powerful Ideas* (Second Edition). New York: Basic Books.

Shaughnessy, J. M. & Burger, F. W. (1985). *Spadework Prior to Deduction in Geometry.* Mathematics Teacher, September 419-428.

Tizard, B. & Hughes, M. (1984). *Young Children Learning.* London: Fontana.

Van Hiele, P. (1986). *Structure and Insight.* United Kingdom: Academic Press, INC.

Van Hiele, P. (1999). *Developing Geometric Thinking through Activities that Begin with Play.* Teaching Children Mathematics, 310-316.

Vygotsky, L. S. (1962). *Thought and Language* (E. Hafmann & G. Vakar). Cambridge: The M.I.T. Press. (Original work published 1939).

Wells, G. (1985). *The Meaning Makers.* London: Hodder & Stoughton.