Hyperbolas and chimneys in classroom

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Abstract

In a second course of Mathematics, Engineering students must learn about integral calculus and its applications. Usually teachers show formulas and examples from the text book, unfortunately those examples have no meaning at all for students. So we have tried to design some didactic activities that let students to build objects with a specific form.

Figure 1. Model of a hyperbolic chimney (used in nuclear power plants) made by a student.

We asked students to build a chimney for a nuclear power plant. They did not have any restriction on the kind of materials they would use only for the form. Students used paper, wires, plastic, aluminium foil, plasticine and other materials to model the shape of a hyperboloid of one sheet. They had to calculate the equation by taking measures from a picture we gave them. In this work we show photos and videos of students' chimneys based on the equations they did.

Keywords

Solid of revolution, volume, area of surface of revolution, hyperbola, integral.

Introduction

In many universities Engineering Programs cover mathematics courses that include calculus in one real variable. Most times it is divided in two courses, the first for differential calculus and the second course that covers integral calculus. In the Instituto Tecnológico y de Estudios Superiores de Monterrey (ITESM) all engineering students cover three courses: Mathematics I - Ma1011 (Differential Calculus), the second course is named Mathematics II – Ma1012 (Integral Calculus) and the third is Mathematics III – Ma1013 (Multivariable calculus).

Our main goal is to involve students in the use of mathematics not just solve routine problems but to apply mathematics in dairy life and in an Engineering context. Teachers are encouraged to use different didactic techniques like Project Based Learning (PjBL), Problem Based Learning and Case Study Methodology.

Theoretical Framework

Our didactic activity was designed under the constructionism theory and following the PjBL technique. Papert (1996) said “Don’t worry if the questions are trivial and repetitive” (p. 38) talking about software and parents, we can talk about routine problems in a classroom and students *don’t worry if exercises are boring and repetitive if they are mathematics*.

In Martin (1996) we can read

> According to constructionist learning theory, people learn most effectively when they are involved in the creation of an external artifact in the world. This artifact becomes an “object to think with,” which is used by the learner to explore and embody ideas related to the topic of inquiry. (pp. 297-298)

Following constructionism we decided our students should build something to relate school with every day life, based on integral calculus and other mathematics courses.

In Project Based Learning web page we can find a precise definition:

> “PjBL is an instructional approach built upon authentic learning activities that engage student interest and motivation. These activities are designed to answer a question or solve a problem and generally reflect the types of learning and work people do in the everyday world outside the classroom” (Buck Institute for Education, n.d.)

PjBL characteristics can be summarized in a short list as

- Lerner-centered environment
- Collaboration
- Curricular content
- Authentic tasks
- Multiple expression modes
- Emphasis on time management
- Innovative assessment

Following PjBL technique we decided that our students should apply analytic geometry and integral calculus to adjust real data from a chimney in Spain and to find the equation of an hyperbola.

Joining both approaches we can say that our design involves the construction of a chimney (using s scale 1cm:10m) by using analytic geometry and integral calculus, letting students to freely choose any mathematical method or technique and any material for the model they need to build.

Activity

Justification

The main idea of this activity is to involve students in the use of "school mathematics" in an engineering context. Students usually learn about circumferences, parabolas, ellipses and hyperbolas and solve exercises like "Find out the center and radius of a circumference that…" or "Given the equation of a hyperbola find out the center, vertices…" and sometimes teachers solve an example of analytic geometry applied to engineering.

In ITESM we try to solve as many real world problems as possible, so the Chimney Activity is an open problem where students must build a hyperbolic chimney following the equation they find based on a photograph. We thought students would feel motivated to search, in internet and mathematics books, for different methods to solve the problem. The activity was used in two different groups with a total of 63 students in an online format; they had two weeks to finish all their work. We wanted the students to globalize their knowledge, that is, students should apply all the mathematical knowledge they have in order to solve the activity.

Activity in detail

We show students different kinds of chimneys and we discuss the many advantages and disadvantages that every kind of chimney has. In the case of a hyperbolic chimney we present the following diagram and photograph where some explanations are given.

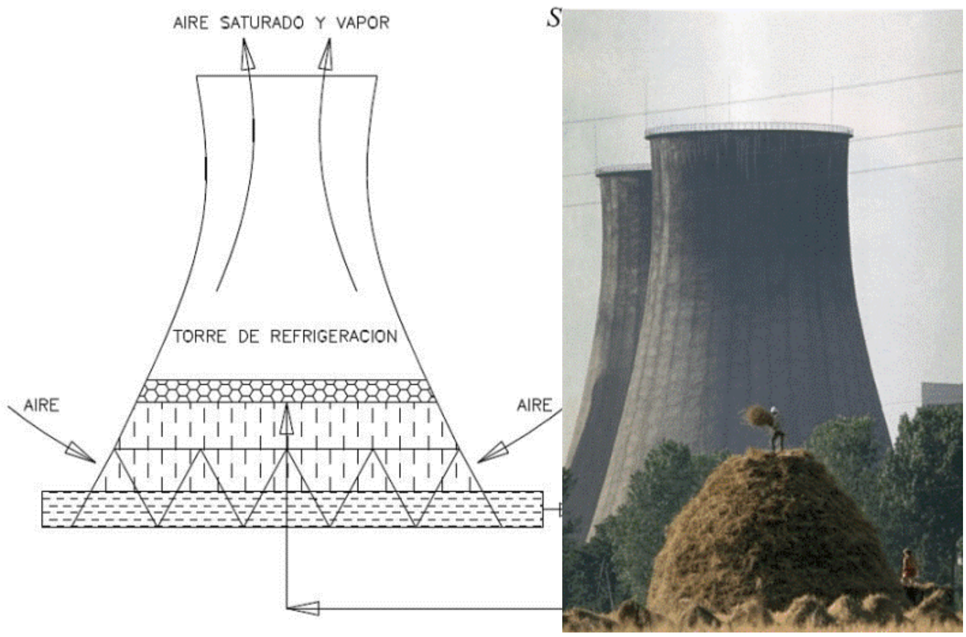

Figure 2. Diagram and actual photograph of a cooling system using a hyperbolic chimney.

Then we gave students the following schema with some measures of a real chimney:

Total height 119m (390.5ft),

Base width 108m (354.3ft),

Top width 75m (246ft),

Mid point width 65m (213.2ft),

Mid point height 88m (288.7ft)

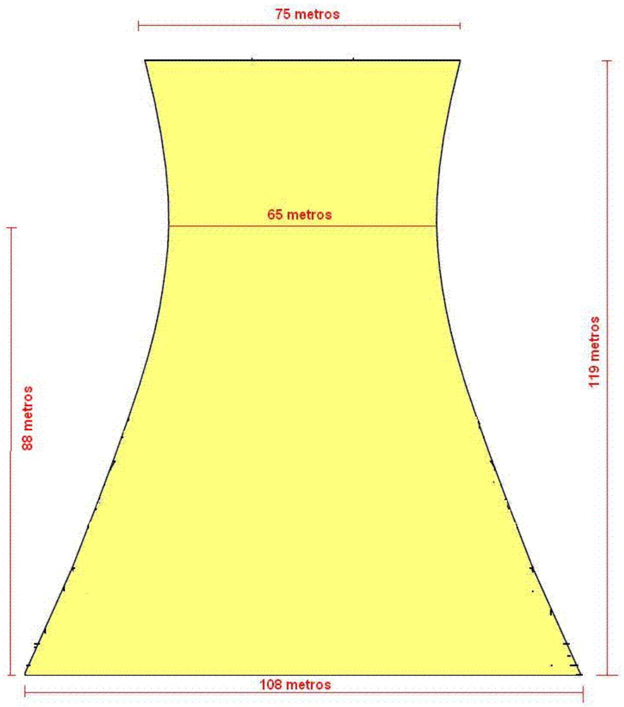

Figure 3. Measures of the chimney in metres.

Then students had the following tasks:

1_ Find out an equation for the hyperbola that generates the chimney.

2_ Find out the volume of the solid of revolution generated by the equation of (1)

3_ Find out the area of the surface of revolution generated by the equation of (1)

4_ Build a scaled model of the chimney using any material you want to use.

Students Answers

We analyzed the answers in two parts, we called Mathematical Part to the first one and deals with the use of mathematics to adjust chimney data with a hyperbola equation, the second one deals with the materials and technique students used to build the model, we called this one the Model Part.

Mathematical part

Some students applied analytic geometry, they drew Cartesian axes system with origin at the center of the hyperbola in figure 3, and then they calculated three or more points to get a system of equations involving the coefficients of the desired hyperbola equation. Solving the equations they were able to give the desired equation. One of these solutions can be seen in figure 4.

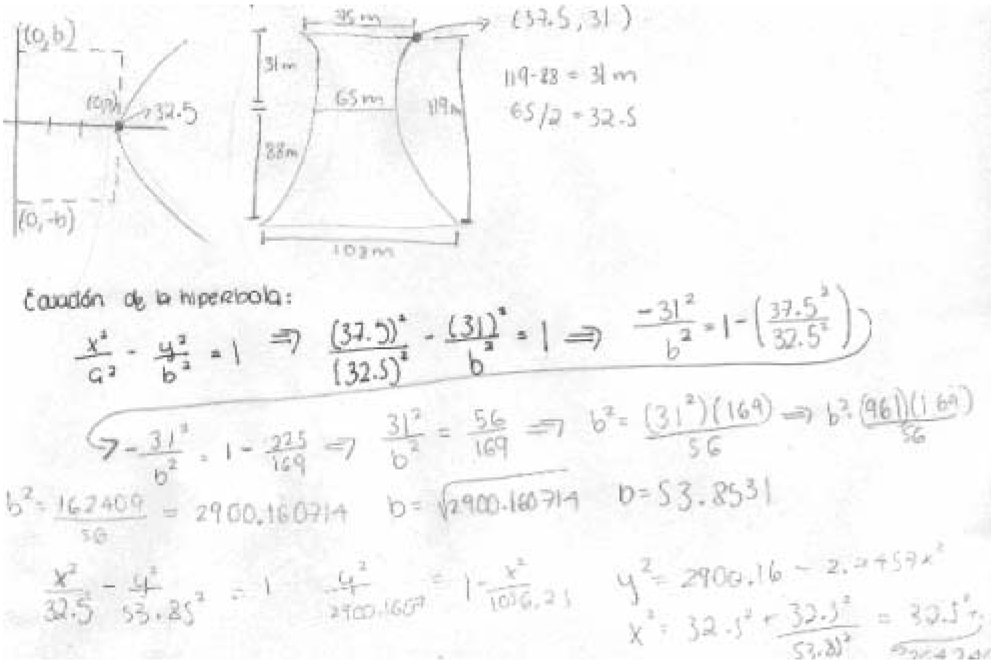

Figure 4. Student's procedure to find the hyperbola equation.

One student found his equation but he tested it and he discovered that the chimney should have to be built using two different hyperbolas. Then he calculated one hyperbola for the upper half and other equation for the lower half of the chimney. This was an unexpected result. Some students drew a vertical line in the middle of the schema in figure 3; they measured it and calculated many points. Then they used specific mathematical software to fix data with a polynomial function.

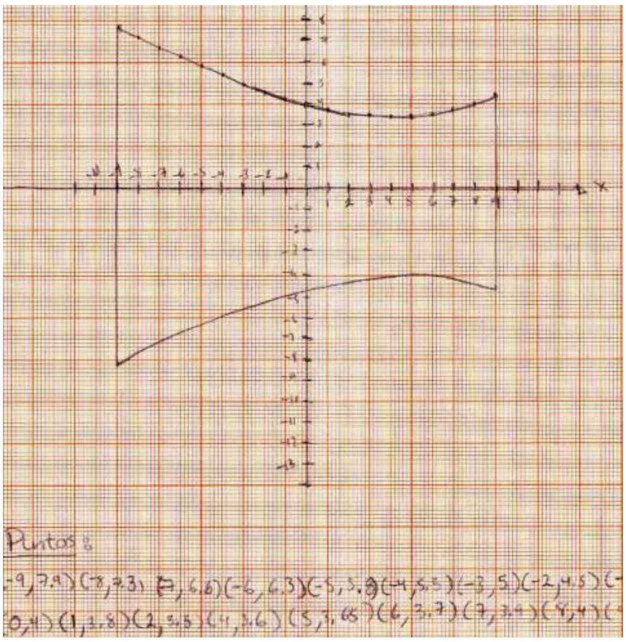

Figure 5. A student's solution included to measure many distances and use them in Mathematica® to fix data.

Every student applied the standard formulas to calculate the volume of the solid of revolution and the surface's area of the solid of revolution with integral calculus. Although they got different equations they got similar results for the volume and the surface's area.

The model

We found two important facts about the model construction, one deals with the material used in the model and the other deals with the shape of the hyperbola. Some students used paper and glue, some used wood, some used wire and paper and other students used a kind of plasticine to build the body of the chimney. Every kind of material had its difficulties.

To get the specific shape of a hyperbola was harder but students found many way. A few students cut cardboard rings with different radius according to the hyperboloid generated by rotating the hyperbola's equation then they covered the rings with plasticine (first chimney in figure 6). Some students used copper wire to cut the hyperbolas and then they welded with circumferences of different radius to get the correct shape, later they covered with paper or tape (second chimney in figure 6). One student searched bottles, cans and vases until she found a can with the correct form; finally she covered the can with plasticine (last chimney in figure 6).

Some students had many troubles to get the exact shape of the chimney, they tried to use only paper and glue but the structure was too heavy and the chimney collapsed. Other students tried only plasticine or other soft materials but chimneys also collapsed. Finally some other tried copper wire (not welded) and adhesive tape (like scotch tape) with the same results.

Figure 6. Chimneys made of cardboard, copper wire and wood.

Conclusions

Two of the main principles in ITESM are "students have to search the knowledge by themselves" and "students have to learn to learn". Our students searched in books and internet for different ways to find a hyperbola equation. We think this was a successful part of the activity because students faced a challenge and they could solve it; and the mentioned principles were fulfilled.

We got many interesting results but our main conclusion is that students really got involved in the construction of the model, they used many different materials for the body of the model, but the most amazing was the richness of different ways they used to get the perfect shape.

The answers we got lead us to think that students are capable to solve some real problems even if they think they can not. We think that this activity was very successful but we know it can be better and we are actually redesigning it.

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