

Application of a Creative Approach by Building Spatial Mechanical Models in a Microworld of Dynamic Geometry

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Abstract

Constructionism principles of education employing computers are often associated with programming. However, there are many environments based on the similar approach applicable in various school subjects. In the field of mathematics, there is a set of dynamic geometry software available. It can be conceived as a microworld which could join school mathematics and computer graphics together. When supported by creative and attractive activities it could be used for learning by doing and at the same time for applying mathematics knowledge. One of the activities is represented by creating movable models using geometric construction tools.

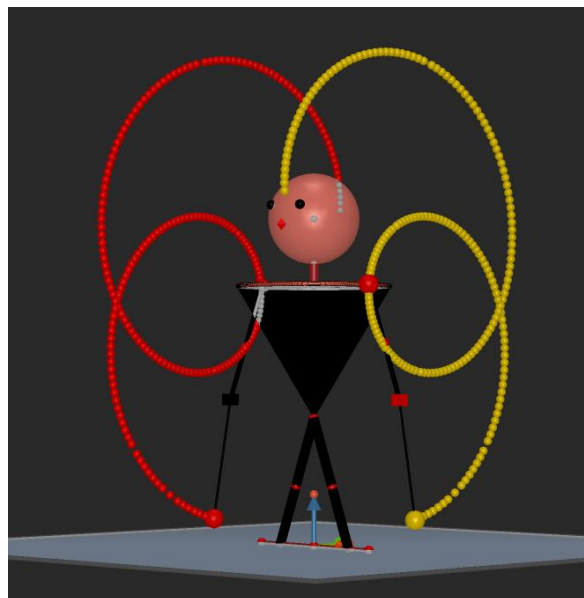


Figure 1. POI dancer and traces of twisting balls on twines, a dynamic geometry figure.

The article deals with a project of modelling simple mechanisms in spatial dynamic geometry software Cabri 3D within a preparatory course for in-service mathematics teachers. The research was to investigate whether the learners were able to leave traditional mathematics teaching approach in order to learn geometry through creating animated models. Three ways of creating computer animation are shown onwards, as well as the principle of a motion in dynamic figure using knowledge of geometry acquired previously is explained. Four difficulty levels of used motion mechanism in a created figure are described. And other experiences referring to the advantages and the difficulties of this activity are commented.

Keywords

Dynamic geometry, mechanisms, modelling, Cabri 3D, pre-service teacher preparation

Dynamic geometry microworld

Dynamic geometry environment represented by Cabri, Geogebra, Cinderella, Sketchpad and other software products, has been recently getting into Czech schools. First experience of using this software as a learning facilitator shows that teachers use computers mostly as a tool for teaching common geometry topics by a standard “paper and pencil” method. A method which uses this software only as a tool for precise and fast drawing is most common among Czech teachers (Vaniček, 2009, p. 78). At the same time, these tools promise much more. They fulfil parameters of a microworld in which students can understand given rules quickly, they can discover, create, project and at the same time they learn mathematics.

Constructionist approach is mainly connected with Logo and other programming environments built on a similar principle. A naturally arising question is whether the educational principles and their results as well as contribution to education proved and realized in Logo might be transferable to any other environment intended for different issues. If the principles of constructionism are similar, the crucial point in constructionist learning depends on the method applied more than the tool used. This article deals with creating computer graphics in another way than usual.

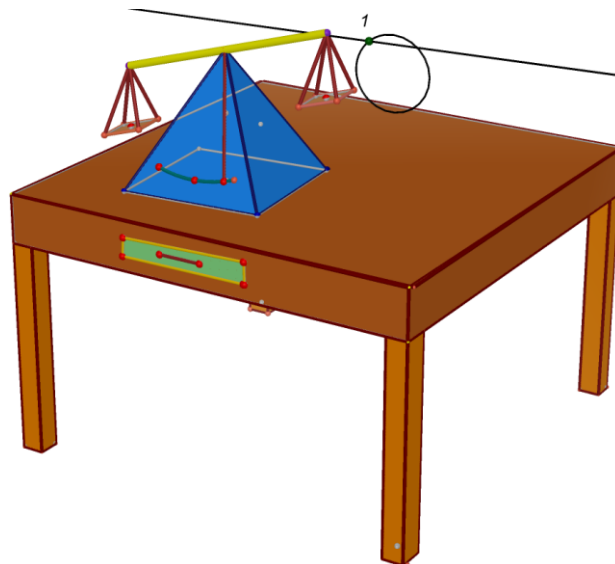


Figure 2. A scale on the table. The internal mechanism of pendulous scale-beams is shown in black (point 1 is moving continuously along the circle and end point of the lever follows the horizontal line).

Dynamic geometry environment software enables learners to work as in a microworld in which they can create graphics by applying geometric knowledge. Their skills that have been learned in classical geometry and improved by constructing figures on the computer can be applied when creating movable models of real objects. For example, it can be a simple engine or any equipment used for moving people or vehicles. Working on open-end tasks can attract students who may feel like programmers while performing. There are two main advantages of this method, a quick progress in learning based on the previous knowledge in geometry and the enriched environment enabling geometric, constructing and project skills development.

Animated graphics creation

One of the benefits of programming in Logo is the graphical output. In addition, turtle drawing comprises of turtle motion itself; some of Logo environments, e. g. Imagine, enable to

comprehend turtle as a moving object (Blaho, Kalaš, 2003). According to Freudenthal (1983, p. 342) motion is, first of all, something that occurs to an object in a space and in a time. It is natural that we can see a motion from different points of view enabling us to think of different ways how to describe moving objects. On the contrary, classical paper-pencil geometry approach understands geometrical figures as sets more than objects and motion in geometry is sometimes seen statically as a parametric geometry.

We can split animated graphics creation on the computer among three basic approaches which nature is different (see Figure 3): direct access, programming and (geometrical) constructing. Although there are more simple approaches for dynamic modelling, the advantage of constructing consists in learning mathematics by doing and giving an interesting direction of applying school knowledge in meaningful and driving activities.

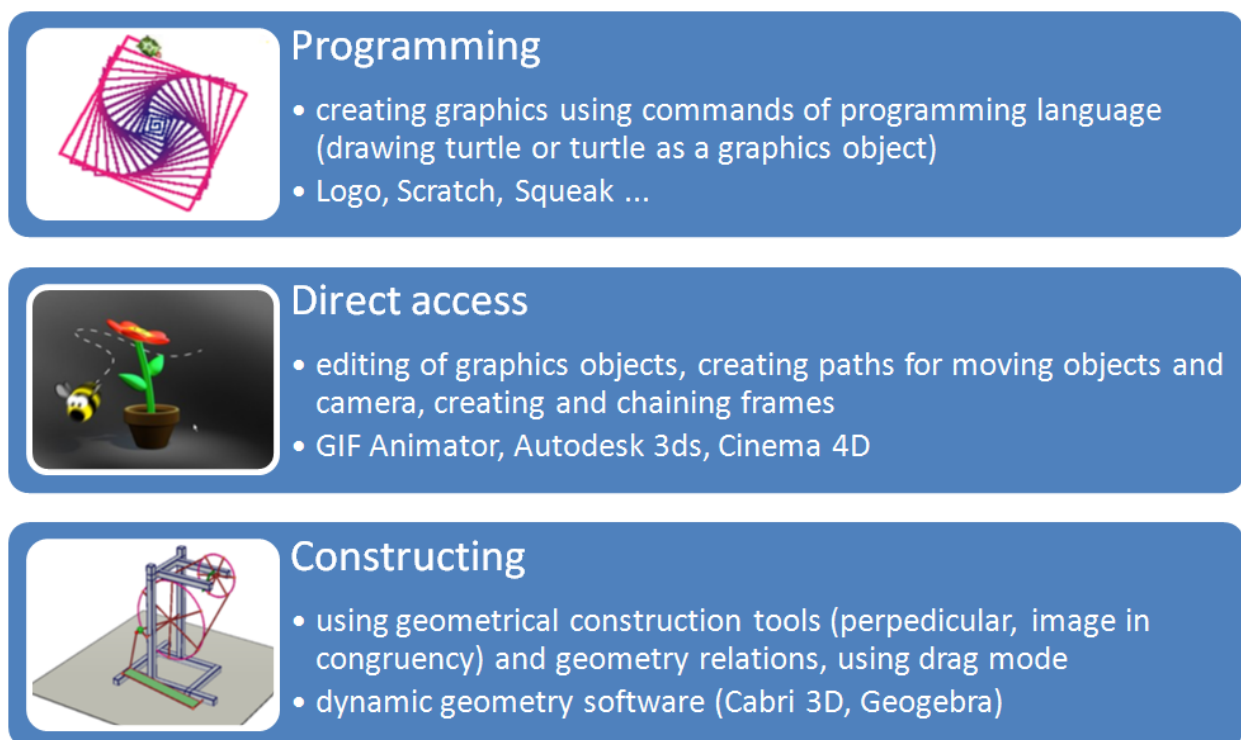


Figure 3. Three types of graphical animation creation on a computer¹

Motion in dynamic geometry figures is realized mostly by so-called “drag modus”. The user can use a mouse to drag suitable point lying e. g. on a line or circle. When dragging this point along the object lying on the other objects in the figure. They change their positions and shapes according to the relationships between objects. E.g. when a line is going through a moving point it has to go through it even when it is manipulated. If this line changes its slope, another line perpendicular to it has to change its slope, too. A task for the author of the dynamic model is to arrange objects and geometrical relationships among them so that changes in positions and shapes look as a real movement. The principle is the same when using plane or spatial geometry software.

Some dynamic geometry software provides automatic animation of chosen points by a special animation tool. There are some other tools using dynamics as Trace and Locus which “stamp”

¹ Second picture downloaded from: Gudang tutorial et 3Dstuff, <http://gudang3d.wordpress.com>, 10.1.2010

chosen moving object to the plane and create a new object from a trace. This is the way of creating complicated curves or surfaces in a geometric figure (see Figure 1).

The fundamentals of work in dynamic geometry are very different from creating graphics in Logo. Final models are not robots as no programming structures (alternative boxes, cycles) are used. Algorithms of geometric construction are completely different from algorithms of turtle behaviour. Nevertheless, we are convinced that this environment allows the same learning style in which, “part of an activity the learner experiences as constructing a meaningful product”, in Papert’s words (Award#8751190).

Mechanical models principles

One of the main features of the dynamic geometry environment is the ability to create movable and interactive pictures using geometric objects that can be dragged with a mouse or moved automatically along a given path (Vrba, 2000). Models constructed in dynamic geometry software are simplified models simulating the main function of a movable object. Functional testing and simulation of a dynamic model is carried out either manually in the drag mode or by the automatic animation (which has the advantage of keeping the observer undistracted). (Schumann, 2004, p. 17).

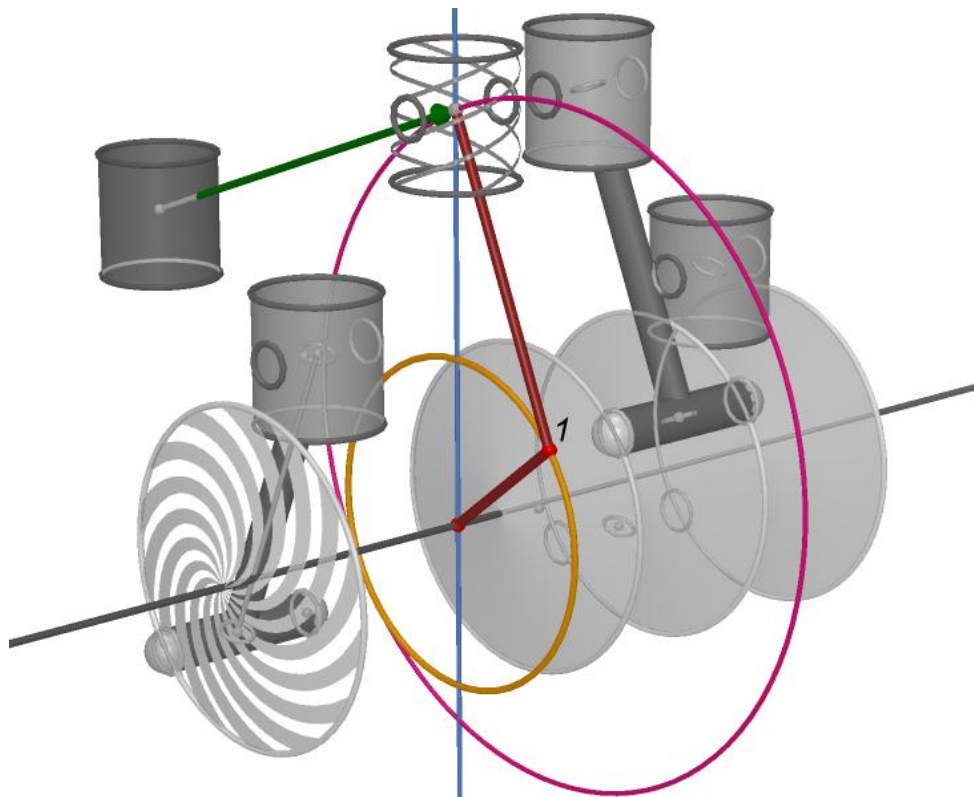


Figure 4. Internal geometry mechanism of an engine model constructed in Cabri 3D, a student’s work. Point 1 is moving along a yellow circle, the piston centre is an intersection point of the vertical blue line and the pink circle which ensures constant length of piston rod. The piston (with small hatches on its surface) is an image of a previously created dark cylinder on the left in translation by green vector.

Users can see an internal geometry mechanism which can move the whole figure by dragging one point. Then the users can create graphic figures without knowing any special commands or language unlike other computer applications, such as graphics and movie editors (GIF Animator, Cinema 4D) or programming environments (programming languages). Dynamic geometry allows

children to apply skills and knowledge previously learned and related to geometry and elementary mechanics in constructing movable models of real things or situations (e.g. simple engines, motion of figures, animals or vehicles). This activity enables us to compare the computable representation of geometry properties to the real mechanism motion by means of dynamic drawings (González-López, 2001).

What do we understand by simple mechanisms? External appearance or theme does not matter, but internal geometry mechanism is important as it controls movement of the others objects while dragging some points. This mechanism is based on clear geometry relations among objects (perpendicularity, parallelity, laying on, projection) and on the dependence among dynamic figure elements. Actually, this mechanism is often hidden in the figure.

One of the examples describing the internal mechanism is a parallel wire on the front and the rear wheel of a bicycle. If the wheels are of the same size they wheel the same velocity and always need to keep parallelism. The rear wheel could be made as an image of the front wheel in translation because parallelism retains in translation. Another example (Figure 5) describes two cogwheels turning in opposite directions. They might be constructed using reflection.

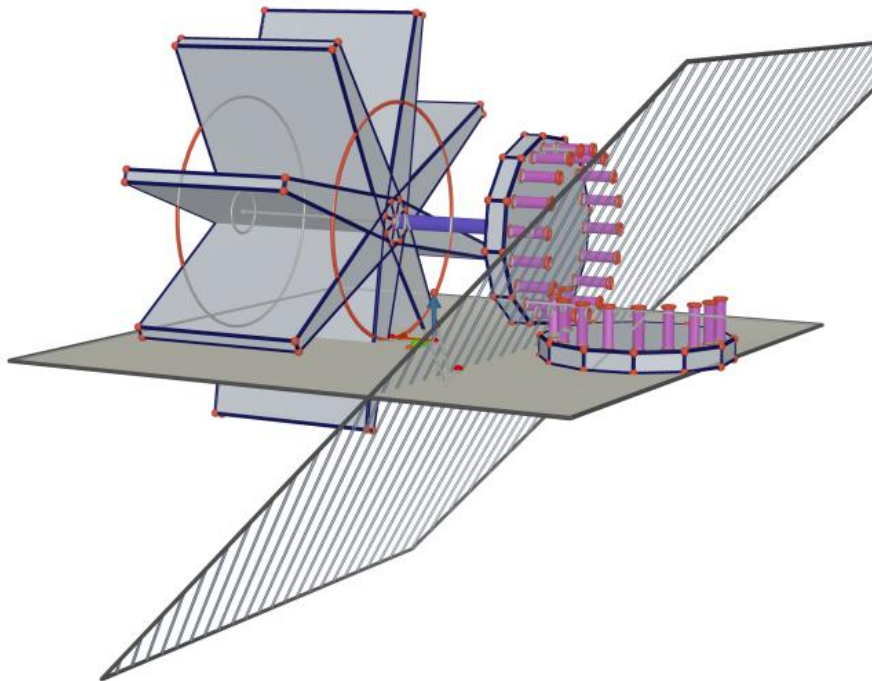


Figure 5 – A model of a water mill made in Cabri 3D. Plane symmetry as a construction step: horizontal transmission wheel is an image of the vertical one with respect of a shaded slant plane.

Spatial geometry modelling

Modelling using dynamic geometry software as a method of teaching geometry is described in various papers dating back to the 90s'. Such open-ended environments as Cabri offer a large range of geometric tools for creating dynamic geometry figures that can be fully animated (Laborde, 1999). Laborde (1996), González-López (2001), Schumann (2004) and others used Cabri for reconstruction a set of simple dynamic models of moving elements and machines described by Bolt in his known book "Mathematics meets Technology" (1991).

Spatial modelling has been limited by non-existence of an appropriate environment till lately. Schumann (2003, p. 14) alludes to three restrictions of modelling in dynamic geometry software, the two of which have been topical up to now: first, confinement to geometrically-describable objects and machineries and secondly, confinement to means and methods of dynamic geometry. The third restriction to objects and machineries which can be interpreted as “plane” has expired due to quick progress in development of dynamic environment for 3D geometry. Of course, most of construction steps used for creating 3D models are still static or “2-dimensional”; however, some of the new tools available are to use e.g.: plane symmetry as seen in the Figure 5.

Levels of difficulty of the motion

Dynamic constructions of mechanical models can be sorted by dependency of moving objects to four difficulty levels (according to Vaníček, 2009, p. 139):

Level No. 1 – **independent motion**

Objects move constantly on basic lines (straight lines, circles), some parts of the figure move independently of one another (using several independent moving points).

Level No. 2 – **dependent but not exact**

There is a central object in the construction. Usually it is a point which controls movement of the other objects that are dependent on it and they move according to the central object. The internal mechanism of the figure does not reflect the reality truly. Sometimes an incorrect mechanism is used, which only looks like a correct one (Figure 6, 7).

Level No. 3 – **geometrically exact**

It is the same principle as in the level described above but the internal mechanism of the figure reflects the reality correctly (Figure 4).

Level No. 4 – **complicated motion applying intersection**

Some movable objects are constructed as dependent on some intersection points. If such intersection point exists then the dependent object exists. This technique enables the author to shift visibility of two similar objects upon an actual situation, so that it looks as if there is only one object with a complicated motion in the figure (Figure 8).



Figure 6. A model of a dancing doll in different stages of the motion and from different perspectives.
A student's work.

A figure of a dancing doll

Let us describe the difficulty level of the dynamic figure representing a dancing doll (see Figure 6). It is a student's project. The doll periodically bends its knuckle and moves both of its forearms, the animation looks perfect. The whole mechanism is created in the way that a movement of one point (No. 1 in the Figure 7) can set the other movable objects move.

The analysis of the hidden mechanism allows us to find out whether every part is constructed correctly. On the phased figure on the left, you need to know, the big red point 1 is constantly moving along the short cyan arch which is a part of the magenta circle. Point 1 is translated by a solid vector to point 2 and point 3 (an elbow) is created as a vertex of a solid triangle with a fixed point 4. The green triangle remains fixed when in motion.

In the two figures on the right, there is a visible mistake. Points 3 and 2 (knee and foot) are moving along the congruent arches, which is incorrect. Both thighs and calves change their length during the motion of point 1. If point 3 is laid on the circle with the middle point 5, the thigh length would be constant. The difficulty of the construction of the doll is only level 2 because it is not exact geometric construction and no advanced technique is used.

It is necessary future teachers could discover, analyze and discuss these mistakes so that they could deeply understand pieces of knowledge applied and later tutor their pupils in such learning activities.

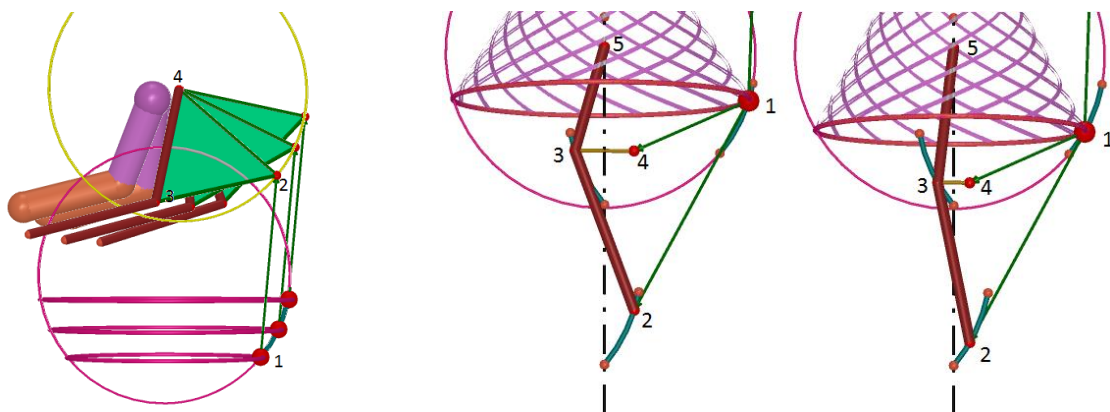


Figure 7. The internal mechanism of the higher (left) and the lower (right) part of the dancing doll. It is evident that the length of both "leg bones" is not constant.

Project of future teachers preparation

When starting to deal with dynamic geometry environment, students meet various difficulties. On the contrary to programming in Logo, which both turtle graphics and work style are completely new to them, the environment of dynamic geometry looks only as a similar to a paper-pencil learning environment. Apparently familiar environment however it has its own specifics fixed in geometry itself. E.g. a point is not mentioned as a place in space but an object. Consequences of this fact are: there could be more than one point positioned on one place, a point lying on a line is not in the same relationship as one to a line going through a point. Moreover, spatial construction uses different construction steps which students do not master as they can when drawing on a sheet of paper.

Other difficulties could be expected from the rank of typical tasks. Student is used to get a problem solving tasks, with an aim exactly described and search for an appropriate algorithm. Creating models seems to be a bit strange to the students, somehow out of topic.

The main reason for realizing such project is to prepare teachers for tutoring such learning activities. The project of creating spatial dynamic models of mechanisms within the preparatory course for future teachers of mathematics was realized at the Faculty of Education, University of South Bohemia this year. We started this project after several years of providing similar project in plane geometry. 17 students experienced in using Cabri 3D took part in this project, most of them had no programming skills. The first part took 7 hours, when students were supposed to create dynamic models in “plane” Cabri first and then they were asked to create a moving figure in Cabri 3D. These “plane” exercises were e.g. creating a rotating prism and a pyramid, a moving engine, a cyclist on a bicycle, a dancing house.

Students seemed to enjoy creating static models, the problem arised when they had to transfer the motion of a point to another part of the figure using construction tools. Students often chose a construction which only evoked correct behaviour of the moving objects, in result some “solid” objects got shorten or disappeared. From a programmer’s point of view, it was not such a big problem when students discovered a geometry procedure which described the reality somehow.

Some of the students’ models were very elaborate, they contained a lot of objects. For example, the model of a dancing doll contained 43 visible and 118 invisible objects, model of railway gates (Figure 8) contained 99 visible and 289 invisible objects. But we claim that quantity of used objects caused less trouble to students than mastering the essentials of internal geometrical mechanism. Most of objects are simple to create, they are static or made in the same way.

We acquired some experience allowing us to conclude it is possible to claim that spatial modelling is much more difficult for students than plane modelling for two reasons:

- Handling the Cabri 3D application is difficult as mere “plane manipulation” with a mouse controls spatial manipulation with both objects and camera. There is no appropriate positioning tool which could facilitate the user to orient in space behind the screen where the construction proceeds;
- Students do not have any experience with 3D construction tools as until recently they have constructed only projections of spatial models into a plane; spatial constructing requires animating spatial imagery which is difficult.

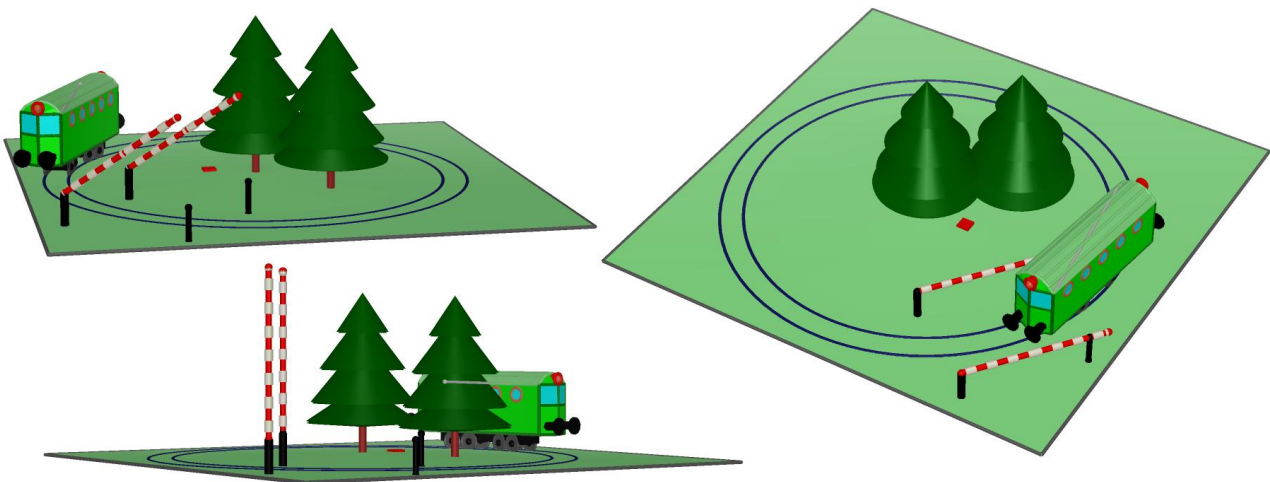


Figure 8. A train and railway gates, a student's work. The gates suddenly start to move when the train is coming closer. The problem of different actions of the gates dependent on the train position (going up and down, staying up or down) is solved by creating three different objects representing the gates which visibility depends on the existence of some hidden intersection points. The model represents difficulty level 4. In these three views, you can see the same train, the same railway and the same trees but three different pairs of gates.

Conclusion

We can conclude that it is worth to use spatial dynamic geometry environment to learn how to create animated models and at the same time direct students to deeper understanding of geometrical concepts which are included in dynamics. Final figures were a good source of discussing the character of geometrical rules and concepts. Such activities suggest how to explain what a creative experiment is in a comprehensive way, which is suitable for trainee teachers who are not able to program.

According to our experience, only less than a half of participating students can enthuse over creative character of these activities, they have to be extra motivated to work creatively. After finishing quite well and correctly behaving figures, some students expressed that they did not enjoy doing the activity. The fact that a part of students do not like creating at all was a bit surprising finding.

Selected difficulty level of movement in mechanical model depends on the author's mathematical skills. It is interesting that this level does not correspond to the author's creativity. Very creative figures may often consist of very simple movements. A big part of authors reveal their creative potential more by choosing the topic and drawing plentiful static parts of the figure. Not only pupils but also pre-service teachers cannot imagine first what technique might be used to manage nicely and correctly behaving animated figures. If we omit a nice design of the finished figure it might be the novelty of possibilities offered by spatial modelling that made this activity so attractive to the students.

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