Restructuring Change, Interpreting Changes: The *DeltaTick* Modeling and Analysis Toolkit

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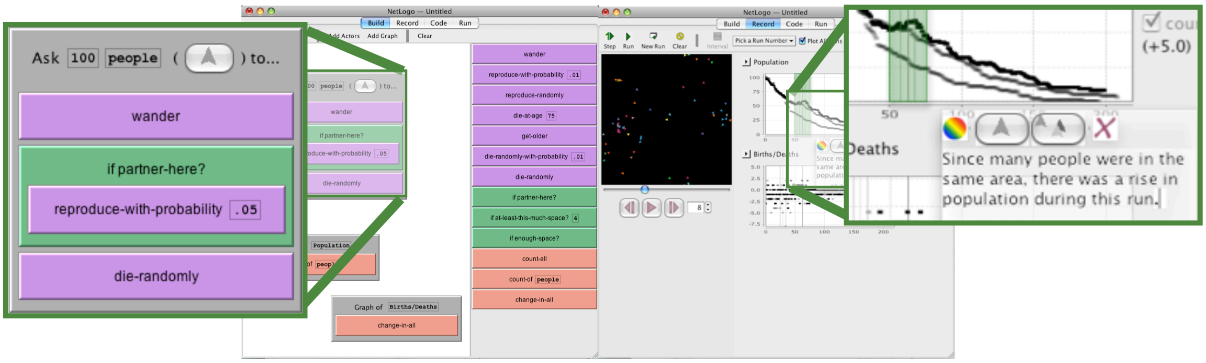
Abstract

Understanding how and why systems change over time is a powerful way to make sense of our world. By modeling those systems, learners have the opportunity to consider how their own actions influence that world, and to make predictions and recommendations for the future. But often, the notion of change is as complex as it is powerful – populations, global temperatures, and economic trends all represent *multiple* events and actors, but are measured in terms of only a few quantities. In this paper, we discuss the motivation and design of *DeltaTick*, an extension to the NetLogo (Wilensky, 1999) agent-based modeling environment that allows learners to easily construct and analyze models of complex quantitative change. To do so, they define models using *agent behavior*-based units, rather than the *rate*-based units typical of equations or systems dynamics models. They can then explore, compare, and annotate model results to investigate how their behavioural models relate to typical equation-based representations.



Since many people were in the same area, there was a rise in population during this run.

Figure 1. Constructing (left) and analyzing (right) models using DeltaTick

Our design is rooted in constructionism (Papert, 1980), and integrates work on complex systems education (Wilensky & Resnick, 1999), low-threshold agent-based modeling (Kahn, 2007; Repenning & Ambach, 1997), representational infrastructure shift (diSessa, 2001; Kaput et al, 2002), and intuitive calculus (Kaput, 1994; Nemirovsky et al, 1993; Stroup, 2002). It leverages what Wilensky and Papert (2006; In Prep) refer to as *restructuration*: re-encoding of disciplinary content with a new representational technology to emphasize different properties of that content. We argue that by constructing and interacting with agent-based models, learners can recognize the *relevance* of ideas of change and variation to learners' own experiences as actors and observers in their world, and the *learnability* of some core concepts of change and variation.

Keywords

mathematics; mathematical modeling; restructuration; agent-based modeling; scientific literacy

Restructuring Change

We want to give learners a way to critically think about how they influence and are influenced by large-scale, systemic changes in their world. *DeltaTick* is a simple, extensible construction and analysis toolkit to support this goal by leveraging recent findings regarding how people think and learn about quantitative change. In this paper, we describe the motivation for the *DeltaTick* environment, discuss some of its design features, and briefly review interviews with learners who used a preliminary version to construct and explore alternative models of population growth. We argue that by providing learners with tools and activities that allow them to express notions of rate and accumulation as the outcome of specific individual behaviors as they occur over time, learners can model and explore quantitative change in a way that (a) emphasizes its relevance to their own lives by leveraging their own experiences of actions and change in the world, and that also (b) provides a novel access point to many of the ideas of mathematical change as they exist in more typical calculus and differential equations-based representations (namely, notions of derivative, integral, and the reversibility of the two).

The theoretical and design contributions of this work are threefold. First, we are exploring a design space that provides learners a low-threshold entry point to building flexible, personally relevant scientific and mathematical models while still having the opportunity (and, as we argue in this paper, encouraging) more sophisticated model refinement. Second, we are leveraging and contributing to existing work on learners' experiences and understanding of quantitative change, but in the specific context of change in complex systems, where multiple interactions and events are embedded in only one or a few measured trends. Finally, we are exploring the role of tools for the analysis of student artifacts in constructionist environments – such that those tools and learners' own artifacts serve as a bridge to typical representations of disciplinary content. In terms of practical contributions, we are working toward providing learners with a viable, intellectually honest alternative to symbolic calculus for modeling mathematical change, while at the same time providing a potential bridge to more typical calculus-based concepts. Our goal is to present the mathematics of change as a relevant, accessible, and empowering tool that can help learners understand and predict their world.

Motivation

Examining rates of change over time and their accumulations have become some of the most ubiquitous practices in not only the natural and social sciences (AAAS, 1991), but also for navigating modern society (Roschelle et al, 2000; OECD, 2006). Often, however, the quantitative trends used to explore economic, environmental, or social phenomena reflect large-scale, systemic processes that involve and affect a number of actors and events. In this sense, it is not just understanding quantitative change, but also understanding how that change reflects the events and interactions of a given system that helps us to make sense of the world and our role as citizens within it.

Shifting representational infrastructures - and specifically, computational tools that represent and simulate processes over time - reflect a powerful way of exploring, thinking about, and simulating change over time - and potentially, for allowing more people to do so (Papert, 1980; diSessa, 2001; Kaput et al, 2002). Agent-based modeling (ABM; Langton, 1997; Wilensky & Resnick, 1999) is one example of a computational representation appropriate for modeling complex systems such as those described above. This technology models a phenomenon by encoding the behaviors and interactions of individual agents or elements of a system (for instance, the rules that govern motion and collision of particles in a gas), and then simulating that system by having a collection of those agents execute those behaviors over time (for instance, to illustrate how that gas exerts pressure on a container; Wilensky, 2003). It has fundamentally changed *how* scientific content is represented and explored, as well as *who* can author and interact with that content (Blikstein & Wilensky, 2009; Levy et al, 2004; Sengupta & Wilensky, 2008).

But while building and interacting with agent-based models can help learners develop a more deep and generative understanding of traditionally advanced content, less is known about how they link this understanding with more conventional representations of those concepts - namely, algebraic and calculus-based equations. This project explores how ABM can serve as an access point to the mathematical aspects of complex phenomena and the ways they connect with the mechanisms and patterns those mathematics represent. To do so, we leverage Wilensky and Papert's (2006; In Preparation) notion of a *restructuration*: a re-encoding of existing disciplinary knowledge using a new representational technology that emphasizes different aspects and properties of that knowledge. In other words, we are exploring how agent-based modelling can be used to provide learners with a new language to "speak" and practice quantitative modeling.

In the following sections of this paper, we describe in more detail the notion of *restructuration*, and make the case for how agent-based modeling can provide learners with more access points to not only specific scientific content, but also to the mathematical representations typically used to present that content. Next, we describe a set of computational tools to provide learners the opportunity to build and explore agent-based models with explicit focus on how those models represent mathematical change over time; along with a short description of the sort of activities that would give learners the opportunities to use these tools meaningfully and constructively. Finally, we discuss some of the specific design features of this environment in the context of preliminary interviews with learners using earlier versions of these tools. We argue that these findings suggest that constructing and analyzing agent-based models with specific attention to ideas of change and variation in systems helps learners to understand the *relevance* of ideas of change and variation in their own lives, as well as makes many difficult concepts in change and variation (such as the reversibility of rate and accumulation) more *learnable* for learners. We conclude with a brief discussion of future work and implications.

The Computational Restructuration of Mathematical Modeling

We base our motivation for the design of DeltaTick within the framework of *restructuration theory* (Wilensky & Papert, 2006; In Prep.) – that different technologies can encode the same disciplinary content, but in ways that emphasize very different aspects and properties of that content. While some *structurations* or encodings of knowledge – for example, using mathematical or agent-based representational systems – are more or less appropriate for some goals or make certain content more accessible and usable, we contend that each can also complement and inform and understanding of the other. Below, we use a figure to illustrate how agent-based modeling (and more generally, computational behavior-based simulation) can be viewed as a restructuration of the ideas of change and variation (boxes 1 and 2), and how it can inform more typical rate-based representations by providing an opportunity to *coordinate* the results of each through the plots or numerical results they generate (boxes 2 and 3). Although the figure is informed by work on mathematical modeling (Niss, et al, 2006) in the sense that a "real situation" is distilled and then formalized into some symbolic notation, we heavily adapt it here to reflect that those situations can be differently conceptualized, that different conceptualizations can be more or less commensurate with a given symbolic encoding, and that symbolic encodings can be mathematical or computational. We are also careful to note that while we are highlighting the connections that are emphasized through the activity of modeling, this is not a clean process - connections can be made between or within any world, depending on the similarities recognized and actions taken by the modeler (Pozzi et al, 1996; Noss et al, 1997).

The box to the left represents a "real world" representation or experience of some dynamic system - for instance, trends in unemployment as a student may experience them as he reads a newspaper article, or searches for a summer job. Those experiences and understandings that are viewed as the key elements, events, trends, or patterns for a particular phenomenon of interest (in this case, unemployment) can be considered together as a “situation model”. In the case of unemployment, an individual may think of his own and his friends' experiences in the workplace; but he may also consider a recent history of rising unemployment, or of national or international trends in consumer spending. These different ways of conceptualizing the causes and effects of a changing system are more or less appropriately represented by different symbol systems - agent-based modeling, for instance, is more appropriate for encoding individual experiences and interactions; while a differential equation or system dynamics model is more appropriate for considering larger-scale patterns and historic trends. Each restructuration, however, can generate results that can be compared and coordinated with one another – so that encodings in one structuration (the specific circumstances that lead to an individual getting or losing a job) can be compared to those in another (increases and decreases in employment levels), and the relationships between them interrogated (as more people are hired, they are able to spend money, which in turn allows more companies to hire more employees).

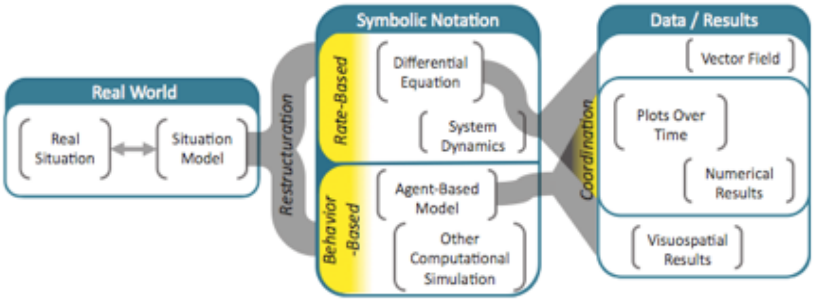


Figure 2. Restructurations are alternate encodings of the same content, with each encoding emphasizing different aspects of that content.

As a result of this shift, agent-based modeling encodes and reflects quantitative change in a way that includes a clear link to specific real-world behaviors that change can represent, as well as emphasizes notions of randomness, sensitivity to local conditions, non-uniform distributions, and other powerful ideas characteristic of *systems* that are not dealt with in traditional calculus. It also provides learners an easy to way to manipulate that encoding in ways they find interesting. On the other hand, this encoding de-emphasizes many of the powerful aspects of typical calculus-based methods, such as the ability to optimize, quickly apply solutions to new and different contexts or scenarios, or quickly compute specific solutions. We argue, and provide evidence below, that it is the transition between different structurations – including the practice of building models in each in order to explore and resolve conflicts between them – which is where a lot of learning can happen around the mathematics of change. In this sense, the plots and numerical results produced by each serve as a *bridging tool* (Abrahamson & Wilensky, 2007) for access to and from typically advanced mathematical and computational concepts.

The DeltaTick Modeling Toolkit

The main goal of our project is to provide learners an easy way to *construct* models of changing real-world systems, and then to *analyze* those models with specific attention to one or a few quantities that are typically used to represent that change.

Constructing Models

To build a model, learners begin by defining one or more types of *actors*, a collection of homogeneous entities that all behave similarly. A window on the construction screen represents each actor type. Learners can then add to those actor windows one or more pre-specified *behaviors* that each actor of that type will execute during each unit of simulated time or “tick”. Behaviors can also be placed inside of *conditions*, which limit the conditions under which each agent performes that behavior happen. Finally, learners can add one or more *graphs*, also represented by a window, to the screen and add one or more quantities of interest that they wish the graph to feature. Finally, users have the option to move to an “advanced” version of the model by clicking on the “Code” tab that allows them to view and modify the text-based NetLogo code that underlies the visual representation – which allows them much more flexibility and generativity than the visual language alone. In Figure 3 below, a user has constructed a model that will start with 100 “people” agents. These agents will each wander around the world, and during any time unit that they encounter another agent nearby (*if partner-here?*), they will have a 5% chance to reproduce. They also each have a 1% chance of dying during each time unit. The model includes two graphs: one of the total population of agents (*Population*), and one of the number of agents added (born) or subtracted (died) from the system (*Births/Deaths*).

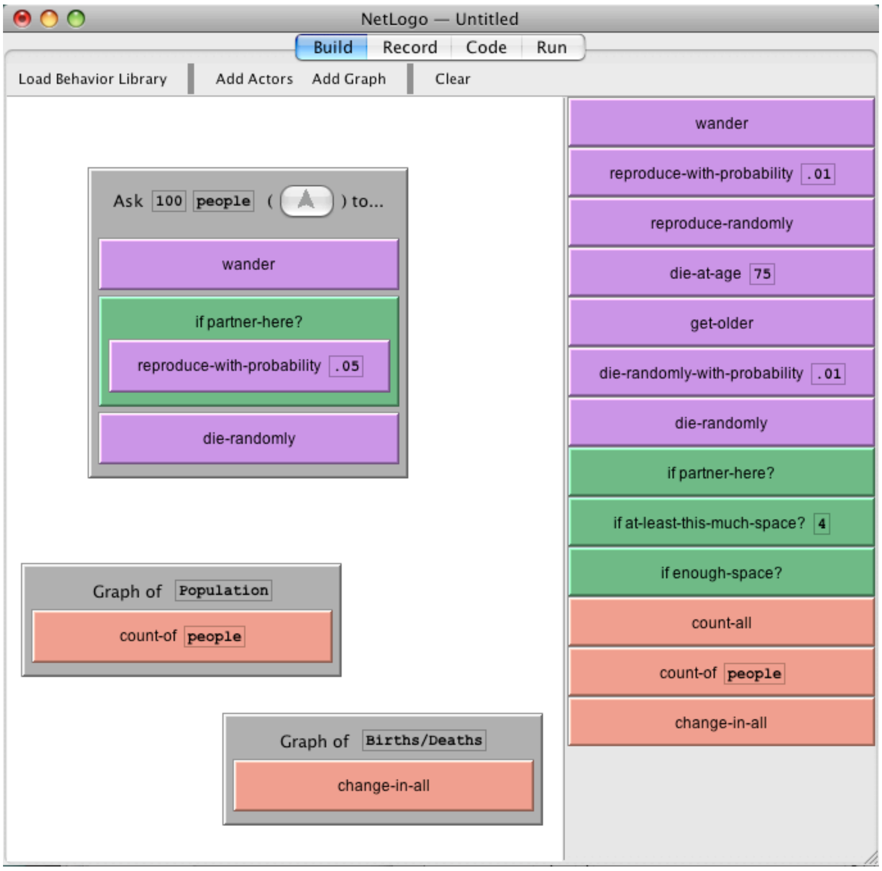
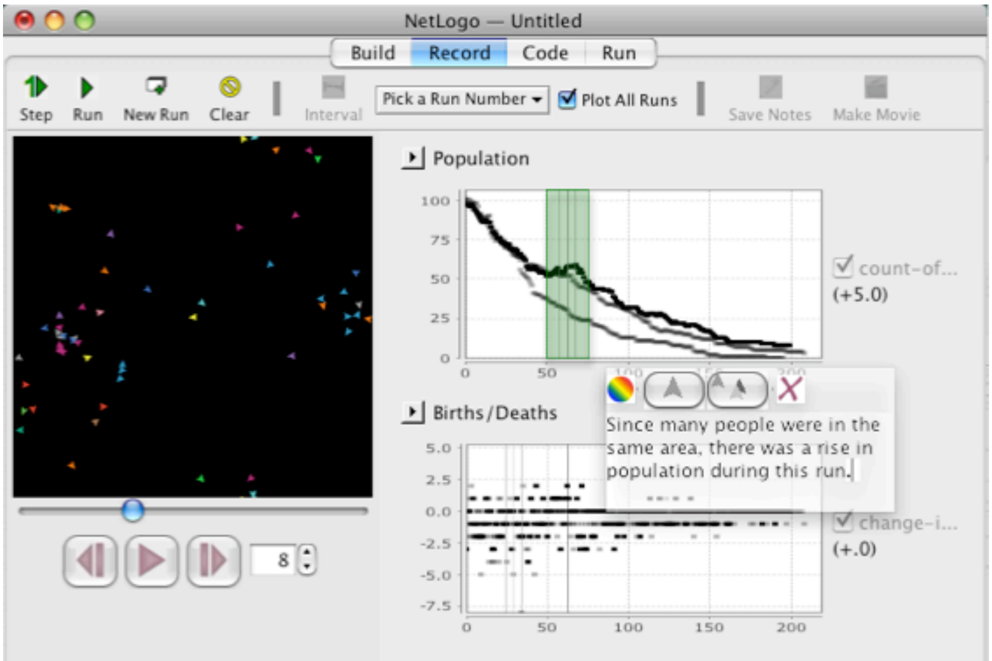


Figure 3. The DeltaTick construction interface. Blocks are designed such that different combinations correspond to markedly different resulting mathematical patterns.

The behavior blocks that appear for learners in this environment are loaded into the environment as sets of “behavior libraries”. Above, a “Population Growth Library” is featured. This library includes individual-level instructions that can affect population growth patters: such as *reproduce-with-probability*, *die-at-age*, or *wander*. It also includes some conditions under which these behaviors might occur for an individual and that might also affect the population growth trajectory: such as *if partner-here?*, or *if enough-space?* (which both affect population growth trends in different ways depending on the density of agents in the world). These libraries are written as xml files that can be imported into the DeltaTick environment, so they are authorable and modifiable. This environment is inspired by Kahn’s microbehaviors (2007) and Repenning’s Behavior Composer (2000), in that behaviors are encapsulated and portable across agents, and that new behaviors can be written and added to the library. However, the behaviors in a given library are be designed for specific disciplinary explorations and to relate to specific potential mathematical patterns, and in that sense are more specific than the behaviors featured in behavior composer, and larger-grained than microbehaviors. This is intended to preserve a more direct relationship between the addition or removal of each behavior block and changes in the resulting mathematical patterns generated by the model, as well as to provide a considerably "low threshold" access point to model construction.

Analyzing Models

After building a model, learners have the opportunity to analyze it using the *HotLink Replay* tool, which includes a visualization of the model and the resulting graphs. These two representations are dynamically linked, so that learners can click on an area of a graph and see its corresponding point in time in the simulation, or play the simulation over time as a cursor indicates the corresponding area on the plot. Learners can also highlight any intervals on a plot, and annotate that interval. In addition to plots and visualizations, the environment also calculates a user-defined piecewise linear approximation of change on any interval of a featured graph. Figure 4 below features three consecutive runs of the model illustrated in Figure 3. The user can switch back and forth between visualizations of each different run of the model; the plot for the specific run that they are visualizing is black and the rest are grey. Below, an interesting feature of the current graph – a point during which the population rose despite a general downward trend – is highlighted in green, and a short annotation is attached to the highlight. The user has also clicked on this interesting point on the graph, so that the visualization itself displays what was going on at that time during the model's execution. Since the model was constructed such that people reproduce only if they find a partner nearby, this plot shows an increase in population while there are clusters of many individual agents together, emphasizing the relationship between specific model rules and the trends that can result from those rules.



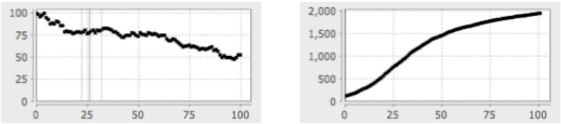
Since many people were in the same area, there was a rise in population during this run.

Figure 4. The HotLink Replay interface. Learners can replay, annotate, and compare different model runs.

HotLink Replay is inspired by environments that have enabled learners to develop more robust understandings of rate and accumulation by providing them a means to *control* a phenomenon that produces change (Kaput, 1994; Wilhelm & Confrey, 2003); interact with *plots* of change and rate of change over time (Confrey et al 1997); and make linkages between *intervals* and *shapes* of plots and the events they represent (Yerushalmy, 1997).

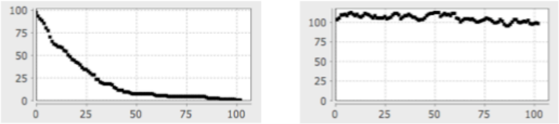
DeltaTick Activities

It is not within a tool, but in a student's use of, interaction with, and discourse around tools and activities where learning happens. As such, we argue the our design provides learners with the opportunity to engage in the activities and ways of thinking that can help them understand change in their world, view it as relevant to their own lives, and learn the powerful underlying concepts that are so ubiquitous in the natural and social sciences, as well as everyday life.



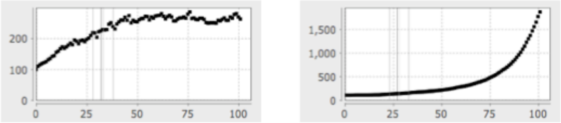
Slow Decline

Logistic-Like Growth



Extinction

Relative Stability



Growth Followed by Stability

Slow, then Rapid Growth

Figure 5. Graph-Matching activities encourage students to consider how different behaviors interact to produce a trend.

Activities we have found to be particularly productive for learners as they interact with this environment are to describe how patterns that do not emerge as a result of conventional mathematical notation can exist in an agent-based model and what those patterns mean (for instance, why the number of births in a population can fluctuate upward and downward even while the overall pattern of growth appears exponential); construct a specific agent-based model that they believe will create graphs that match graphs that we or their peers provide (akin to “my graph rules” activities; Wilensky & Abrahamson, 2006); research topics of interest within the domain of population trends using public scientific data on the behaviors and trends that characterize populations and use that data to create an agent-based model; find mathematical functions that approximate the quantitative trends produced by their own models; and hypothesize how different behaviors in their model correspond (or do not correspond) to different elements of the mathematical models conventionally used to represent population growth. As our collection of behavior libraries grow, different activities may emerge for different domains.

Student Interviews

To explore whether an approach such as the one above was feasible, as well as to explore our hypothesis that agent-based modelling can provide learners a new and productive means to engage with and think about mathematical content, an early, text-based version of the construction tools described above were introduced to 10 U. S. high school learners (ages 15-17) during semi-structured clinical interviews in Summer 2009. In this section we will briefly describe some examples of how learners (a) recognized agent-based modeling a way to explore specific, personally-relevant questions about population growth, and (b) connected mathematical notions of rate, accumulation, and the relationship between the two to model behavior.

Relevance: Meaningful Modeling and Extension

One of the questions we were most interested in was whether building models using a set of prespecified behavior-based units allowed learners to recognize the flexibility of this modeling language and the applicability of notions of change and variation to real-world systems. During our interviews, we encouraged learners to modify a simple, exponential model of population growth we initially provided them (in which agents each simply had a 1% chance of reproducing for each iteration of time) in any way they chose. When given this opportunity, 7 out of 10 learners added behavior parameters or behavior sequences in a way that they explicitly related to real-world behaviors, or included real-world constraints: for instance, learners explicitly mentioned issues of life expectancy (*Interview 7*) and the heterogeneity of life expectancy (*Interview 8*), family planning (*Interview 11*), and the role of age in partner selection (*Interview 3*), or the fact that many factors interact to produce patterns in a "nation or city" (*Interviews 8 and 9*). Many of these factors are ones that are difficult to include in conventional mathematical models.

In addition to constructing models that they find relevant and applicable for thinking about the world, we are also interested in making it easier for learners to think about extensions to their model, rather than only using what is available in the pre-specified library. In our interviews, 4 out of 10 learners explicitly suggested new behaviors they wished to add the their models that were not available, and 3 of those learners actually wrote NetLogo code with the to create new behaviors they could add to their models (no learners had prior experience with NetLogo).

Learnability: Interpreting and Connecting to Conventional Representations

Another question we were interested in was whether learners were able to interact with notions of rate of change, accumulation, and the relationship between the two in a way that lets them "unpack" these notions as they relate to the mechanisms of change in systems. In this section, we recount two such cases.

Rate as Representing Complex, Multi-behavioral Events. We found that often in our interviews, making sense of aggregate rates of change in terms of individual behaviors was an interpretive challenge for learners (much how it is difficult for learners to interpret behaviors at different levels in a systems; Wilensky & Resnick, 1999). In several cases, learners started off speaking of rate and derivative as an inert mathematical notion disconnected from the very phenomena it is intended to model. This was the case for Hannah (*Interview 6*), who before building her own models was asked what might make the graph rise, then fall. She suggested that "a genocide or natural disaster happened", but went on to explicitly note that events that reduce the population "doesn't really affect the rate, it just, it's just something like an outside thing that affects the population".

Hannah's confusion regarding what behaviors or aspects of the model the *rate* of population change actually represented was echoed by many other learners – 6 of those interviewed were not able to make a link between the number of people born in the model and the rate of change of the population for a given unit of time without guidance. After having modified the model to observe how different behaviors all contributed to the same quantity and the way it changed over time, however, Hannah not only felt comfortable talking about rate in the context of more than one behavior, but also in terms of how differences in how a single agent behaves (extending agents' life span) can interact with other behaviors (more time to reproduce) and contribute to overall changes in population over time: "Well I knew that if I increased the death age then it wouldn't decrease as much and um, I don't think I needed to increase the uh, probability of the to reproduce… because they would have more time to live and reproduce they would still be people (mmkay) around."

By observing and controlling a *mathematical* idea in terms of *behavior*, we suggest that Hannah was able to integrate an understanding of how multiple behaviors interact with one another with the notion of rate, which measures the results of those behaviors. In this case, she thought about multiple different ways to increase population: first, by giving each agent more time to reproduce by allowing them to live longer, and second, by increasing their probability of reproducing at each tick.

Defining Mathematical Terms with Behavioral Relationships. Finally, we argue that representing change in systems in terms of agent behaviors can provide learners with insight into how the mathematical notions of rate of change and accumulation relate, and what they represent in a modeling context. In one interview, we ask Brooke (*Interview 3*) how we could find the population's rate of change for a given year. Although the model featured a plot of the number of individuals born at each tick in the model, as well as a plot of the total population per tick from which this information could be extracted, she only suggests that "you could use derivatives" – presumably referring to the mathematical procedure for determining a rate of change given an algebraic expression of the change itself – as a means to determine change for a tick in the model. When probed for whether she could think of any other way "with all the information you're given here", she responded "Um, I dunno, I'd have to think about that. Kind of like derivatives all stuck in my mind."

Later, after explicitly being asked how the two plots featured in the model are related, Brooke recognizes birth in the population as defining the rate of change in this simple model: "…our original population is taking this (points to lower graph) added to uh people that there were, that there were beforehand, (mhm) before they were (mhm) the people were born". Later, when asked whether she could relate these graphs to the idea of a derivative, Brooke notes: "…derivatives is basically taking like an exact (mhm) point divided by another exact point finding the exact um, like change, but this gives us the exact change over the exact time. It gives us the exact number of people born at a certain time which is what derivatives is, is solving for."

Discussion and Future Work

Change – especially complex, systemic change – is an increasingly important part of our world. In this paper, we argue that agent-based modelling can provide learners with a new way to "speak change", as well as a bridge to the conventional mathematical models used to represent such change. The results we have reported are preliminary, and we are currently conducting a new series of studies that we hope will provide more insight into the relationship between agent-based and mathematical modeling. However, we are excited by the potential that such environments hold for exposing learners to the complexity, power, and relevance of the mathematics of change for understanding our world.

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